

Pharmaceutical Generics, Vertical Product Differentiation, and Public Policy*

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Abstract

This paper studies oligopolistic competition in off-patent pharmaceutical markets using a vertical product differentiation model. This model can explain the observation that countries with stronger regulations have smaller generic market shares. It can also explain the differences in observed regulatory regimes. Stronger regulation may be due to a higher proportion of production that is done by foreign firms. Finally, a closely related model can account for the observed increase in prices by patent owners after entry of generic producers.

Keywords: Pharmaceutical industry, generics, vertical product differentiation.

JEL Classification: I10, I18, L18, L65

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1 Introduction

Few markets exhibit such extensive price regulation as the market for pharmaceutical products¹. There is evidence that price regulation can actually reduce competition. Market shares of lower-priced (generic) versions of a product tend to be lower in countries with more severely constrained prices. In a recent study, Danzon and Chao (2000) show that the countries with stronger price regulations are usually those where the effect on price of the number of generic producers is smaller.

The purpose of this paper is to explain, first, this tendency of regulation to generate higher market shares for the higher-priced versions of the product, and also to endogenize the choice of policies by the countries. This, in turn, has some policy implications for supra-national organizations, like the European Union.

We do this by proposing a duopoly model of vertical product differentiation in which, first, the government announces a price ceiling. Then, the firms set a “perceived” quality level for the product, and finally they compete in prices, taking into account the government-set price ceiling. We find that the lower the price ceiling, the higher the market share of the higher-priced variety. This is independent of the parameter values of the model: the size and variety of tastes in the market, and the size and convexity of the costs of producing perceived quality. The intuition for this result relies on the fact that market shares depend on the ratio of the price ceiling to the high quality. But the quality responds to the price ceiling less than proportionally, due to the convexity of the cost of quality.

The other important result of the paper has to do with public policy. It is not clear in this context what should be the objective function of the planner. We have been quite careful so far in talking about a “perceived” quality level. The definition of a generic pharmaceutical product varies somewhat from country to country, but in general it is required to be therapeutically equivalent in most key clinical dimensions as the product which it is designed to replace. For this reason one could consider that “perceived quality” should be irrelevant to the regulator, whose objective should merely be to maximize “actual” welfare. We show that under a social welfare function which uses a measure of “objective” welfare for the consumers, the implied policy is

¹“In sum, efforts by national authorities to curb pharmaceutical costs and offset the demand-increasing effects of generous health care insurance by imposing drug price controls are found throughout the industrialized and less-developed world” Scherer (2000).

to set the lowest ceiling consistent with the participation constraint for the firm. This is true even though the firm's profits are incorporated into the social welfare measure. This is, arguably, not what we observe even in the most regulated countries. And, in any case, it cannot account for the cross-sectional variations that we observe between the regulatory environment of different countries. So it seems reasonable to explore alternative measures of welfare.

The alternative we consider is to use the “perceived” utility of the consumers for the consumer surplus. We find that the “optimal” price ceiling, from the point of view of the companies, may be higher than the one that maximizes total social welfare (consumer surplus plus firms’ profits). This is because lower regulated prices imply more competition in quality. When quality production is expensive, this is bad for the companies. Indeed, high marginal costs of production of quality is one of the conditions under which society prefers lower ceilings than the producers.

This result is interesting because countries with stronger regulations (and similar sizes, and wealth) tend to be the largest net importers of pharmaceutical products. Which suggests that regulation could be driven by considering only the welfare of the “local” agents, the consumers. This observation, in turn, leads to the implication that supranational organizations, like the European, could undertake a role in harmonizing the price regulations in a way that internalizes the welfare of all actors involved in this game, thereby possibly enhancing total social welfare. Naturally, there may be a need for supranational compensations to achieve agreements.

Earlier literature on generic competition (Frank and Salkever 1992, 1997, Scherer 1996, pp. 376-378) has emphasized the fact that brand-name drug prices tend to increase after generic entry. The simpler version of our model cannot account for this fact. However, a slightly modified version of the model, presented in appendix A, predicts brand-name price increases after generic entry. The reason for the difference between the models is a conflict between the market segmentation and market reduction effects of generic entry.

2 Background and previous work

An introduction and overview to the issue of price controls in the pharmaceutical industry can be obtained from Danzon (1997). Scherer (2000) has a

section on price controls, besides providing an introduction to the pharmaceutical industry. Jacobzone (2000) is a descriptive work summarizing public policies in the pharmaceutical market.

Danzon and Chao (2000) show that price regulation causes a decreased impact of generic competition in price reductions. Previous empirical work had shown some evidence of competition after generic entry (Grabowski and Vernon 1992, Reekie 1996, Ellison 1997), but the results could not compare such a large array of countries with different regulatory regimes to make informed guesses as to the effect of policies. None of these studies investigates theoretically the causes underlying the difference in regulatory regimes.

One of the more studied regulatory measures is that of reference pricing (López-Casasnovas and Puig Junoy 2001 survey the literature on reference pricing). Mestre-Ferrándiz (2001) studies the impact of reference prices in pharmaceutical markets with generic competition. He studies two different versions of the system. In one version consumers are subsidised a flat sum of money for the product, irrespective of which brand they buy. In another version they are paid a proportion of the final price of the good. He finds that, the first version increases costs for the health authority but enhances consumers' welfare. The second system may actually decrease costs for the health system. This paper is closer to ours than other papers which also model the effect of reference prices in pharmaceutical markets (Danzon and Liu 1997, Zweifel and Crivelly 1997). The others tend to focus on the price impacts, rather than on the welfare and political economy sides of the question. Mestre-Ferrándiz, however, does not try to understand the cross country differences that Danzon and Chao (2000) uncover.

Grabowski and Vernon (1992) shows that generic entry was followed by price increases by the branded producer, a result later confirmed by Frank and Salkever (1997). This was called the Generic Competition Paradox by Scherer (1993). Frank and Salkever (1992) or Mestre-Ferrándiz (1999) provide theoretical explanations for this phenomenon using brand loyalty in horizontal product differentiation models. We show, without using brand loyalty, that vertical product differentiation can also explain this fact, although only for some distributions of tastes. Indeed, Caves, Whinston and Hurwitz (1992), with a different sample show modest price decreases by the branded good producer after generic entry.

Gambardella, Orsenigo and Pammolli (2000) study the issue of innovation in pharmaceuticals. We have neglected this issue because patent length and breadth is supposed to be the tool for innovation policy. But their paper

coincides with ours in emphasizing the need for coordination between policies in different countries. Lack of coordination would lead to (potentially welfare-decreasing) free-riding by the countries which do not house innovating firms.

3 The model

We use a game of vertical product differentiation² to analyze the problem.

Consumers have utility function:

$$U = \begin{cases} \theta u - p, & \text{if she buys one unit of the good of quality } u \text{ at price } p \\ 0, & \text{otherwise} \end{cases}$$

The symbol θ represents a taste parameter. The distribution of θ is uniform and $\theta \in [0, \bar{\theta}]$. The total mass of consumers is given by S .

Firm A is the branded producer and firm B is the generic producer. Firms decide on the quality they want to produce. Quality here should be understood as “perceived” quality, as generic products are legislated to be therapeutically identical to the branded products. To produce different “quality” levels, firms have to alter consumer’s perceptions, which they can do by resorting to marketing tools. So they incur a fixed cost $F_i = k_A \frac{u^\gamma}{\gamma}$, $i \in \{A, B\}$, which is convex in the level of quality.³ We assume that $k_A \leq k_B$ so that the branded producer can produce quality at a lower cost than the generic producer. This is reasonable since the branded producer has typically been the patent holder and has been in the market for a long time, so at a minimum there is less uncertainty about its product.

The game unfolds as follows. First the government “declares” a maximum price that the producers can charge for the product. Then, the firms decide on the quality level, u , with which they will endow their products. Finally the firms compete in prices.

We solve the game backwards.

Denote by \bar{p} the maximum price set by the government.

We first solve for the demand faced by the top and bottom quality firms. Denote by θ_{hl} the buyer indifferent between the high quality and the low quality goods. Then, given the utility function, $\theta_{hl} = (p_h - p_l)/(q_h - q_l)$.

²See e.g. Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and Motta (1993).

³This particular functional form is widely used in this type of models. See, e.g., Motta (1993).

Denote by θ_{l0} the buyer indifferent between the low quality good and not consuming. Then, $\theta_{l0} = p_l/q_l$. Since θ is uniformly distributed, and the mass of consumers is S , we have that the demands are:

$$D_h = \frac{S}{\bar{\theta}} \left(\bar{\theta} - \frac{p_h - p_l}{u_h - u_l} \right), \quad D_l = \frac{S}{\bar{\theta}} \left(\frac{p_h - p_l}{u_h - u_l} - \frac{p_l}{u_l} \right).$$

Assuming that the constraint $p_h \leq \bar{p}$ is not binding, the reaction function for the high quality firm is given by:

$$\frac{S}{\bar{\theta}} \left(\bar{\theta} - \frac{2p_h - p_l}{u_h - u_l} \right) = 0,$$

So that the best response to p_l by the h firm is:

$$p_h = \begin{cases} \frac{\bar{\theta}(u_h - u_l) + p_l}{2}, & \text{if } \frac{\bar{\theta}(u_h - u_l) - p_l}{2} \leq \bar{p} \\ \bar{p}, & \text{otherwise} \end{cases}$$

The reaction function for the low quality firm, taking as given the high quality price, is:

$$\frac{S}{\bar{\theta}} \left(\frac{p_h - 2p_l}{u_h - u_l} - \frac{2p_l}{u_l} \right) = 0,$$

from where we obtain:

$$p_l = \frac{p_h u_l}{2u_h}.$$

So that the equilibrium of this subgame is:

$$(p_h, p_l) = \begin{cases} \left(\frac{2\bar{\theta}(u_h - u_l)u_h}{4u_h - u_l}, \frac{\bar{\theta}(u_h - u_l)u_l}{4u_h - u_l} \right) & \text{if } \frac{2\bar{\theta}(u_h - u_l)u_h}{4u_h - u_l} \leq \bar{p} \\ \left(\bar{p}, \frac{\bar{p}u_l}{2u_h} \right) & \text{otherwise} \end{cases}$$

We first calculate the equilibrium in the quality subgame assuming that the price constraint is not binding, and then turn to the situation with the binding price constraint. We assume that the high quality will be produced by the branded producer and the low quality by the generic producer, and check whether this is an equilibrium. At least for some parameter values the opposite situation (generic producers delivering the perceived higher quality) would also be an equilibrium. Since this is not the empirically relevant case, we will focus on the equilibrium where the perceived high quality is produced by the branded producer⁴.

⁴For additional theoretical and experimental reasons why this is the relevant equilibrium (in a closely related model) see Cabrales, García-Fontes and Motta (2000) and Cabrales and Motta (2001).

3.1 Non-binding price ceilings

We now substitute the expression for the equilibrium prices into the profit functions of the firms:

$$\begin{aligned}\Pi_h &= \frac{4S\bar{\theta}(u_h - u_l)u_h^2}{(4u_h - u_l)^2} - k_A \frac{u_h^\gamma}{\gamma}, \\ \Pi_l &= \frac{S\bar{\theta}(u_h - u_l)u_h u_l}{(4u_h - u_l)^2} - k_B \frac{u_l^\gamma}{\gamma}\end{aligned}$$

The first order conditions for the firms in the quality subgame are:

$$\begin{aligned}\frac{\partial \Pi_h}{\partial u_h} &= \frac{4\bar{\theta}S [4u_h^3 - 3u_h^2 u_l + 2u_h u_l^2]}{(4u_h - u_l)^3} - k_A u_h^{\gamma-1} = 0, \\ \frac{\partial \Pi_l}{\partial u_l} &= \frac{\bar{\theta}S [4u_h^2 - 7u_h^2 u_l]}{(4u_h - u_l)^3} - k_B u_l^{\gamma-1} = 0.\end{aligned}$$

Let $\mu = u_h/u_l$. Then, if we divide marginal revenues by marginal costs we obtain:

$$\frac{4(4\mu^2 - 3\mu + 2)}{4\mu^2 - 7\mu} = \frac{k_A}{k_B} \mu^{\gamma-1}$$

which leads to the following implicit expression for μ :

$$\frac{k_A}{k_B} \mu^{\gamma-1} (4\mu^2 - 7\mu) - 16\mu^2 + 12\mu - 8 = 0$$

$$\text{Let } f(\mu) = \frac{k_A}{k_B} \mu^{\gamma-1} (4\mu^2 - 7\mu) - 16\mu^2 + 12\mu - 8$$

Lemma 1 *The equation $f(\mu) = 0$ can have only one solution for $\mu \geq 1$.*

Proof. See appendix B. ■

The lemma shows that there can only be one possible equilibrium with the brand-name producer as the higher-priced variety. To obtain the candidate equilibrium qualities we go back to the first order conditions and we obtain:

$$u_h^* = \left(\frac{4\bar{\theta}S [4\mu^3 - 3\mu^2 + 2]}{k_A (4\mu - 1)^3} \right)^{\frac{1}{\gamma-1}}, \quad u_l^* = \left(\frac{\bar{\theta}S [4\mu^3 - 7\mu^2]}{k_B (4\mu - 1)^3} \right)^{\frac{1}{\gamma-1}}$$

We have said that this is a “candidate” equilibrium. This is so because the profit functions from where the first-order conditions are obtained assume

that firm A produces the higher quality and firm B the lower one. So there could be, in principle, values for the quality for firm A (respectively B) such that the profits were higher if firm A chose a value of quality lower than u_l^* (or a value higher than u_h^* for firm B) holding u_l^* (respectively u_h^*) constant. We have not been able to show that this is not the case analytically. However, extensive numerical simulations seem to rule it out. As an example, Figure 1 shows the candidate equilibrium profits (calculated numerically with Matlab) for both firms, for a range of values of k_A/k_B and fixed values for the remaining parameters. It also shows the highest possible profits for firm A (respectively B) when choosing a value of quality lower than u_l^* (or a value higher than u_h^* for firm B) holding u_l^* (respectively u_h^*) constant. All our numerical simulations are qualitatively identical. So, at least for all those parameters for which we have checked, the candidate equilibrium is indeed an equilibrium. Notice, too, that it is the only candidate for an equilibrium.

From our characterization of an equilibrium, we can get one result of interest.

Proposition 2 *If $\gamma \geq 2$, the equilibrium price of the high quality good is higher under monopoly than under duopoly.*

Proof. See Appendix B. ■

Earlier literature on generic competition (Frank and Salkever 1992, 1997) has emphasized the fact that brand-name drug prices tend to increase after generic entry. As we just saw, the simpler version of our model cannot account for this fact. Generic entry has two different effects on prices. On the one hand it induces market segmentation. The brand-name producer specializes in selling to consumers with stronger tastes for quality, who are prepared to pay a higher price for the product. This first effect tends to make brand-name prices higher. On the other hand, it reduces the size of the market over which quality expenses have to be recouped. Thus, it reduces the incentives to spend in quality, and so the prices that can be charged. In the simple version of the model we just presented the size effect dominates, so the prices are lower. In an appendix we study a related model in which the segmentation effect dominates. The only change is that the distribution of consumers is more polarized in the modified version. We could pursue the rest of the study with a model in which the segmentation effect could be stronger than in this version, at the expense of computational complexity. Although this would perhaps enhance the realism of the model (and it would certainly

be necessary for precise policy work), we feel that it would unnecessarily obscure the remaining presentation.

3.2 Binding price ceiling

We now turn our attention to the case where the price constraint is binding.

We substitute the expression for the price into the profit functions of the firms:

$$\begin{aligned}\Pi_h &= \frac{S\bar{p}}{\theta} \left[\bar{\theta} - \frac{2\bar{p}u_h - \bar{p}u_l}{2u_h(u_h - u_l)} \right] - k_A \frac{u_h^\gamma}{\gamma}, \\ \Pi_l &= \frac{S\bar{p}^2 u_l}{4\bar{\theta}u_h(u_h - u_l)} - k_B \frac{u_l^\gamma}{\gamma}\end{aligned}$$

The first order conditions for the firms in the quality subgame are:

$$\begin{aligned}\frac{\partial \Pi_h}{\partial u_h} &= \frac{S\bar{p}^2}{\theta} \left[\frac{2u_h^2 - 2u_h u_l + u_l^2}{2u_h^2(u_h - u_l)^2} \right] - k_A u_h^{\gamma-1} = 0, \\ \frac{\partial \Pi_l}{\partial u_l} &= \frac{S\bar{p}^2}{4\bar{\theta}(u_h - u_l)^2} - k_B u_l^{\gamma-1} = 0\end{aligned}$$

Let $\mu = u_h/u_l$. Then, if we divide marginal revenues by marginal costs we obtain:

$$2 \left(2 - \frac{2}{\mu} + \frac{1}{\mu^2} \right) = \frac{k_A}{k_B} \mu^{\gamma-1}$$

which leads to the following implicit expression for μ :

$$\frac{k_A}{k_B} \mu^{\gamma+1} - 4\mu^2 + 4\mu - 2 = 0$$

$$\text{Let } f(\mu) = \frac{k_A}{k_B} \mu^{\gamma+1} - 4\mu^2 + 4\mu - 2$$

Lemma 3 *The equation $f(\mu) = 0$ can have only one solution for $\mu \geq 1$.*

Proof. See Appendix B. ■

The lemma shows that there can only be one equilibrium with the brand-name producer as the higher-priced variety.

The equilibrium values of the high and low quality, expressed as a function of μ are obtained from the first order conditions:

$$u_h^* = \left[\frac{S\bar{p}^2(2\mu^2 - 2\mu + 1)}{k_A\bar{\theta}_i 2(\mu - 1)^2} \right]^{\frac{1}{\gamma+1}}, \quad u_l^* = \left[\frac{S\bar{p}^2}{k_B 4\bar{\theta}(\mu - 1)^2} \right]^{\frac{1}{\gamma+1}}.$$

As before, we can establish numerically that there are no profitable deviations such that A produces the low quality and B the high quality.

We can at this point check the second order conditions for the problem.

$$\begin{aligned} \frac{\partial^2 \Pi_h}{\partial u_h^2} &= \frac{S\bar{p}^2(2u_h - u_l)(-2u_l^2)}{\bar{\theta}} - k_A(\gamma - 1)u_h^{\gamma-2} \leq 0, \\ \frac{\partial^2 \Pi_l}{\partial u_l^2} &= \frac{2S\bar{p}^2}{4\bar{\theta}(u_h - u_l)^3} - k_B(\gamma - 1)u_l^{\gamma-2} = 0 \end{aligned}$$

But for the values that satisfy the first order conditions $\frac{S\bar{p}^2}{4\bar{\theta}(\mu u_l - u_l)^2} = k_B u_l^{\gamma-1}$ so that $\frac{\partial^2 \Pi_l}{\partial u_l^2} = \frac{2k_B u_l^{\gamma-1}}{(u_h - u_l)} - k_B(\gamma - 1)u_l^{\gamma-2}$. So the second order conditions will be satisfied provided that $\mu \geq 1 + \frac{2}{\gamma-1}$ ⁵.

From the characterization of equilibrium we can already establish one of the main results of this paper:

Proposition 4 *The relative market share of the high quality good is a decreasing function of the maximum price \bar{p} , that is, the lower the maximum price, the higher the relative market share of the high quality product.*

Proof. See Appendix B. ■

This result depends on the fact that, in this model, individuals consume only one unit of the good⁶. So relative market shares depend only on the “position” of the individual who is indifferent between the two varieties of

⁵For $\gamma = 2$, this is satisfied as long as $\mu \geq 3$ which is true if $\frac{k_A}{k_B} \leq \frac{26}{27}$. For $\gamma = 3$, this is satisfied as long as $\mu \geq 2$ which is true if $\frac{k_A}{k_B} \leq \frac{10}{8}$. For $\gamma = 4$, this is satisfied as long as $\mu \geq \frac{5}{3}$ which is true if $\frac{k_A}{k_B} \leq \frac{114}{125}$. These values are reasonable cost advantages, but the cost advantage necessary becomes higher when the value of γ is very large, to the point that for $\gamma \rightarrow \infty$, the limiting value of the cost advantage is $\frac{2}{e^2} \simeq 0.26$.

⁶This is actually a good assumption in a market where the amount of consumption is mostly controlled by the physician.

the good. This, in turn, depends on the ratio of the price ceiling to the quality (either one, as the proportion between them is independent of the price ceiling). But the quality responds to the price ceiling less than proportionally, due to the convexity of the cost of quality. Since price is, then, reduced proportionally more than quality when the price ceiling is reduced, the indifferent individual is closer to the producer of the low-priced variety. The effect is actually stronger, the harder it is to modify quality perceptions (the more convex the cost of quality function). It is critical, then, for this result, that companies devote a large proportion of their efforts to product marketing. Indeed, pharmaceutical companies advertising budgets are typically at least as large as those dedicated to R&D.

4 Welfare analysis and public policy

We will discuss the issue of social welfare under two alternative assumptions about what the policy maker takes into account when taking decisions about public policy. One possibility is that the policy maker disregards the quality evaluation of the different consumers, as the branded product and the generic one have to be therapeutically equivalent under most legislations.

In this case, the authority would take into account as the “consumer surplus” a certain constant, K , per consumer that effectively consumes, minus the price paid. So consumer surplus would be: $\widehat{CS} = (K - \bar{p})D_h + (K - p_l)D_l$ and social welfare: $\widehat{SW} = \widehat{CS} + \Pi_h + \Pi_l$. This expression is equivalent to:

$$\begin{aligned}\widehat{SW}_A &= KD_h + KD_l - k_A \frac{u_h^\gamma}{\gamma} - k_B \frac{u_l^\gamma}{\gamma} \\ &= SK \left[1 - \frac{\bar{p}}{2\theta u_h} \right] - k_A \frac{u_h^\gamma}{\gamma} - k_B \frac{u_l^\gamma}{\gamma}\end{aligned}$$

From this expression we can easily get:

Proposition 5 *The social welfare function \widehat{SW}_A is maximized by making the minimum possible price consistent with firms’ participation.*

Proof. Both u_h and u_l are increasing in \bar{p} . Similarly, \bar{p}/u_l is also increasing in \bar{p} . Thus, \widehat{SW}_A is decreasing in \bar{p} . ■

Although this would appear to be a sensible policy, taking into account the “objective” properties of these drugs, we do not observe regulators typically using price caps in such a strong way that it drives pharmaceutical

companies to their reservation utilities. So, even if this form of regulation looks normatively sensible, the regulators' behavior cannot be well explained with this type of social welfare function in the context of this model.

There is an alternative form of regulation which can better explain behavior. If regulators care about the "perceived utility" of the consumers, then the consumers' surplus would take a different form. After all, the regulator is (or is appointed by) an elected politician, and voters probably use their "perceived utility" when voting, rather than the "objective" therapeutical value of the drug.

In this case, consumer surplus will be:

$$\begin{aligned} CS_h + CS_l &= S \left[\int_{\theta_{hl}}^{\bar{\theta}} (\theta u_h - \bar{p}) \frac{1}{\theta} d\theta + \int_{\theta_{l0}}^{\theta_{hl}} (\theta u_l - p_l) \frac{1}{\theta} d\theta \right] \\ &= \frac{S u_h}{2\bar{\theta}} [\bar{\theta}^2 - \theta_{hl}^2] - \bar{p} D_h + \frac{S u_l}{2\bar{\theta}} [\theta_{hl}^2 - \theta_{l0}^2] - p_l D_l \end{aligned}$$

Lemma 6 1. *The sum of profits can be expressed as:*

$$\Pi_h + \Pi_l = S\bar{p} - \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{(\mu-1)(4\mu-3)}{\mu} + \frac{k_A \mu^\gamma + 1}{\gamma} \right].$$

2. *Let $SW = CS_h + CS_l + \Pi_h + \Pi_l$, we have that:*

$$\begin{aligned} SW &= \frac{S\mu\bar{\theta}}{2} \left(\frac{S\bar{p}^2}{k_B 4\bar{\theta}(\mu-1)^2} \right)^{\frac{1}{\gamma+1}} - \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{2\mu^2 - \mu + 1}{\mu} + \frac{k_A \mu^\gamma + 1}{\gamma} \right] \\ &\quad - k_B^{\frac{2}{\gamma+1}} \left(\frac{S\bar{p}^2}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma-1}{\gamma+1}} \end{aligned}$$

Proof. See Appendix B. ■

$$\text{Let } A = \frac{S\mu\bar{\theta}}{2} \left(\frac{S}{k_B 4\bar{\theta}(\mu-1)^2} \right)^{\frac{1}{\gamma+1}}, B = \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{2\mu^2 - \mu + 1}{\mu} + \frac{k_A \mu^\gamma + 1}{\gamma} \right], C = k_B^{\frac{2}{\gamma+1}} \left(\frac{S}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma-1}{\gamma+1}} \text{ and } D = \left(\frac{S k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{(\mu-1)(4\mu-3)}{\mu} + \frac{k_A \mu^\gamma + 1}{\gamma} \right].$$

Proposition 7 *If $1 < \gamma \frac{B}{A} \left(\frac{S}{\frac{2\gamma}{\gamma+1} D} \right)^2 + (\gamma-1) \frac{C}{A} \left(\frac{S}{\frac{2\gamma}{\gamma+1} D} \right)^{\frac{2(\gamma-2)}{\gamma-1}}$, then the optimal \bar{p} for SW is smaller than the optimal \bar{p} for $\Pi_h + \Pi_l$.*

Proof. See Appendix B. ■

This proposition is a bit hard to interpret since it holds under a condition that depends on the parameters in a complicated way. To understand this condition it is useful to see that:

$$\gamma \frac{B}{A} = \frac{2\gamma k_B^{\frac{2}{\gamma+1}}}{S\mu\bar{\theta}} \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma-1}{\gamma+1}} \left[\frac{2\mu^2 - \mu + 1}{\mu} + \frac{k_A\mu^\gamma + 1}{\gamma} \right]$$

So that

$$\gamma \frac{B}{A} \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^2 = \frac{4(\mu-1)^2(\gamma+1)^2 \left[2\mu^2 - \mu + 1 + \frac{k_A\mu^{\gamma+1+\mu}}{\gamma} \right]}{2\gamma \left[(\mu-1)(4\mu-3) + \frac{k_A\mu^{\gamma+1+\mu}}{\gamma} \right]^2}$$

And from this we have a couple of corollaries that are easier to interpret.

Corollary 8 *There is a value of $\gamma < \infty$, large enough that the optimal \bar{p} for SW is smaller than the optimal \bar{p} for $\Pi_h + \Pi_l$.*

Proof. See Appendix B. ■

In words, this says that for high γ , companies do not like low price ceilings. Under these conditions qualities are very expensive to produce, so they will be close to each other. Thus competition in prices will be quite strong, even without government intervention. Introducing binding price ceilings in this context may be very detrimental to firms.

Corollary 9 *If $2\mu^3 + 7\mu^2 - 58\mu + 12 > 0$, and $\gamma \geq 2$, there is a value of $k_A > 0$, small enough that the optimal \bar{p} for SW is smaller than the optimal \bar{p} for $\Pi_h + \Pi_l$.*

Proof. See Appendix B. ■

Here the branded producer has noticeably lower costs of quality than the generic producer. For this reason differentiation is cheap and profitable, so firms can locate in comfortable market niches with substantial monopoly power. The introduction of price ceilings can disrupt this arrangement.

There will be other environments where price ceilings are not so bad for firms. If they have similar and low costs of producing quality, they may compete as hard in that dimension as they would in the price dimension, with bad results for the competitors. So they would favor some external

force constraining them, whereas the consumers would be happier just letting them compete strongly.

However, given the evidence that countries with stronger regulations (even those with similar sizes, and wealth) tend to be the largest net importers of pharmaceutical products, it is reasonable to think that one of the corollaries hold. Given the early mover advantage that the branded producer enjoys, it seems most likely that corollary 8 holds. Further empirical work would be necessary to fully answer the question.

5 Conclusions and further work

This paper has studied price regulations in an oligopolistic market with vertical product differentiation. We feel this is an appropriate model for the pharmaceutical market when generic products compete with brand name ex-patent holders. The conclusions from the model can explain the empirical evidence available from this market. It also provides some implications for policy work.

We have not studied the connection of off-patent markets with the markets during the time that the patent holds. It would be interesting to investigate the connection between the two periods. In principle the incentives to innovation should come from the duration of the patent period and its breadth, but from the point of view of welfare it could be better to connect the patent policy with the post-patent regulatory environment.

Another aspect that we do not study is the regulation of pharmacies and the prescription activity by physicians. A more complete model should include all those actors, and the incentives provided for their activities.

But we feel that the more rewarding area for future work would be the empirical investigation of determinants of policies across countries. We have emphasized that the importance of foreign versus national production of pharmaceutical products is key in determining the regulatory regime. Although there is some evidence in that direction, a more careful study of that, and other aspects influencing regulation, would be necessary.

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6 Appendix A

In this appendix we propose a variation of our model which can account for the fact that the producer of the high-priced variety charges a higher price under competition than under monopoly. As in the main text we have that consumers have utility functions: $\theta u - p$, if she buys one unit of the good of quality u at price p , and 0 otherwise. But now the distribution of θ is not uniform, but rather has a two point support. A proportion p of the population has a taste parameter $\bar{\theta}$ and a proportion $(1 - p)$ has a taste parameter $\underline{\theta}$. The total mass of consumers is given by S . The fixed cost of quality level u is as before, $F_i = k_A \frac{u^\gamma}{\gamma}$, $i \in \{A, B\}$.

The game unfolds as follows. The firms decide on the (unique) quality level, u , with which they will endow their products⁷. Then the firms compete in prices.

First, we analyze the game under monopoly. We will later turn to the duopoly case.

Suppose first that the firm decides to serve both consumer types. We will check later under which conditions this is optimal. If both types of consumer are served, then the price will be $p_m = \underline{\theta} u_m$ so that profits are $\Pi_m = S \underline{\theta} u_m - k_A \frac{u_m^\gamma}{\gamma}$. Then $\frac{\partial \Pi_m}{\partial u_m} = S \underline{\theta} - k_A u_m^{\gamma-1}$, which implies that $u_m = \left(\frac{S \underline{\theta}}{k_A}\right)^{\frac{1}{\gamma-1}}$, $p_m = \underline{\theta} \left(\frac{S \underline{\theta}}{k_A}\right)^{\frac{1}{\gamma-1}}$ and $\Pi_m = \left(\frac{S \underline{\theta}}{k_A}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\gamma-1}{\gamma}\right)$.

Suppose now that only the high type of consumer is served., then the price will be $p_m = \bar{\theta} u_m$ so that profits are $\Pi_m = S \bar{\theta} p u_m - k_A \frac{u_m^\gamma}{\gamma}$. Then $\frac{\partial \Pi_m}{\partial u_m} = S \bar{\theta} p - k_A u_m^{\gamma-1}$, which implies that $u_m = \left(\frac{S \bar{\theta} p}{k_A}\right)^{\frac{1}{\gamma-1}}$ and $\Pi_m = \left(\frac{S \bar{\theta} p}{k_A}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\gamma-1}{\gamma}\right)$. The condition for the market to be served entirely is then $\bar{\theta} p \leq \underline{\theta}$ or $\frac{\bar{\theta} p}{\underline{\theta}} \leq 1$.

Let us analyze now the duopoly case. In the price game the low price firm will charge $p_l = \underline{\theta} u_l$. The high price p_h has to be set so that the low quality firm does not want to attract the consumers with higher preference for quality. To do that, the low quality firm would have to set p^* such that $\bar{\theta} u_l - p^* = \bar{\theta} u_h - p_h$ so that $p^* = p_h - \bar{\theta}(u_h - u_l)$. Notice that this implies that $p^* < p_h - \underline{\theta}(u_h - u_l)$ which implies that $\underline{\theta} u_l - p^* \geq \underline{\theta} u_h - p_h$ and low types do not want to go the high quality firm if p^* is offered. If the low quality firm

⁷This assumes that monopolies cannot price discriminate by creating two varieties with different quality levels and prices. This could be because the fixed cost of brand creation is too large to be justified by the price discrimination benefit. In fact we do not typically observe more than one product variety offered under monopoly.

sets p^* its profits in the price subgame (remember that the quality costs are sunk at this stage) are $S p^*$. So to avoid this deviation p_h has to be at most such that $S p^* = S(1-p)\underline{\theta}u_l$, which implies that $p_h = \bar{\theta}(u_h - u_l) + (1-p)\underline{\theta}u_l$.

Now we can solve the quality subgame. $\Pi_l = S(1-p)\underline{\theta}u_l - k_B \frac{u_l^\gamma}{\gamma}$. Then $\frac{\partial \Pi_l}{\partial u_l} = S(1-p)\underline{\theta} - k_B u_l^{\gamma-1}$, which implies that $u_l = \left(\frac{S(1-p)\underline{\theta}}{k_B}\right)^{\frac{1}{\gamma-1}}$. $\Pi_h = S p \left(\bar{\theta}(u_h - u_l) + (1-p)\underline{\theta}u_l\right) - k_A \frac{u_h^\gamma}{\gamma}$. Then $\frac{\partial \Pi_h}{\partial u_h} = S p \bar{\theta} - k_A u_h^{\gamma-1}$, which implies that $u_h = \left(\frac{S p \bar{\theta}}{k_A}\right)^{\frac{1}{\gamma-1}}$. Thus we have that:

$$p_h = \bar{\theta} \left(\left(\frac{S p \bar{\theta}}{k_A} \right)^{\frac{1}{\gamma-1}} - \left(\frac{S(1-p)\underline{\theta}}{k_B} \right)^{\frac{1}{\gamma-1}} \right) + (1-p)\underline{\theta} \left(\frac{S(1-p)\underline{\theta}}{k_B} \right)^{\frac{1}{\gamma-1}}$$

The previous discussion can be summarized in the following

Proposition 10 *The price of the high quality variety under duopoly is higher than the price under monopoly (that is, $p_h \geq p_m$) if:*

$$\bar{\theta} \left(\left(\frac{S p \bar{\theta}}{k_A} \right)^{\frac{1}{\gamma-1}} - \left(\frac{S(1-p)\underline{\theta}}{k_B} \right)^{\frac{1}{\gamma-1}} \right) + (1-p)\underline{\theta} \left(\frac{S(1-p)\underline{\theta}}{k_B} \right)^{\frac{1}{\gamma-1}} \geq \underline{\theta} \left(\frac{S \underline{\theta}}{k_A} \right)^{\frac{1}{\gamma-1}}$$

Let $\lambda = \frac{\bar{\theta}}{\underline{\theta}}$ and remember that to have the monopolist catering to the whole population we need $\lambda p \leq 1$.

Corollary 11 *If $\gamma = 2$, then the price of the high quality variety under duopoly is higher than the price under monopoly (that is, $p_h \geq p_m$) if:*

$$\lambda p \geq \frac{\lambda \frac{k_A}{k_B} + 1 - (1-p)^2 \frac{k_A}{k_B}}{\lambda + \frac{k_A}{k_B}}$$

Proof. From the proposition we have that if $\gamma = 2$, $p_h/p_m \geq 1$ when

$$\lambda^2 p - \lambda(1-p) \frac{k_A}{k_B} + (1-p)^2 \frac{k_A}{k_B} \geq 1$$

A straightforward manipulation of the expression leads to the result. \blacksquare

The condition in the corollary reduces to $\lambda p \geq \frac{\lambda+1-(1-p)^2}{\lambda+1} = 1 - \frac{(1-p)^2}{1+\lambda}$ when $\frac{k_A}{k_B} = 1$.

7 Appendix B

Proof of Lemma 1

First note that $f(\mu) < 0$ for $\mu \leq \frac{7}{4}$, so that any solution has to be larger than $\frac{7}{4}$. We will now show that to the right of $\frac{7}{4}$, the function f is first decreasing and then increasing. Since $f(\frac{7}{4}) < 0$, and the function has to be eventually positive, the result will follow.

Let μ^* be the lowest value of $\mu \geq \frac{7}{4}$ such that $f'(\mu^*) = 0$. Since $f'(\frac{7}{4}) < 0$, this implies that $f'(\mu)$ must be increasing at $\mu = \mu^*$. Thus $f''(\mu^*) > 0$. Also, notice that f' is a convex function to the right of $\frac{7}{4}$. Thus, for $\mu \geq \mu^*$, we have that $f'(\mu) \geq f'(\mu^*) + f''(\mu^*)(\mu - \mu^*) = f''(\mu^*)(\mu - \mu^*) \geq 0$, so that $f'(\mu) \geq 0$, for $\mu \geq \mu^*$. Since μ^* is the lowest value of $\mu \geq \frac{7}{4}$ such that $f'(\mu^*) = 0$, then $f'(\mu) \leq 0$, for $\frac{7}{4} \leq \mu \leq \mu^*$ and $f'(\mu) \geq 0$, for $\mu \geq \mu^*$.

Proof of proposition 2

The price under monopoly, is given by $p_m = \frac{\bar{\theta}}{2}u_m$, and the optimal quality $u_m = \left(\frac{S\bar{\theta}^2}{k_A 4}\right)^{\frac{1}{\gamma-1}}$. This implies that $p_m = \left(\frac{S\bar{\theta}^{\gamma+1}}{k_A 2^{\gamma+1}}\right)^{\frac{1}{\gamma-1}}$. Under duopoly $p_h = \frac{2\bar{\theta}(\mu-1)u_h}{4\mu-1} = \frac{2\bar{\theta}(\mu-1)}{4\mu-1} \left(\frac{4\bar{\theta}S[4\mu^3-3\mu^2+2]}{k_A(4\mu-1)^3}\right)^{\frac{1}{\gamma-1}}$. This implies that

$$\frac{p_h}{p_m} = \frac{4(\mu-1)}{4\mu-1} \left(\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1}\right)^{\frac{1}{\gamma-1}}.$$

Since $\mu \geq 1$, we have that $\frac{4(\mu-1)}{4\mu-1} < 1$. So, if $\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1} \leq 1$, then $\frac{p_h}{p_m} < 1$. If $\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1} > 1$, then $\left(\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1}\right)^{\frac{1}{\gamma-1}} \leq \frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1}$, since $\gamma \geq 2$ so $\frac{p_h}{p_m} \leq \frac{4(\mu-1)}{4\mu-1} \left(\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1}\right)$. But $\frac{4(\mu-1)}{4\mu-1} \left(\frac{64\mu^3-48\mu^2+32}{64\mu^3-48\mu^2+12\mu-1}\right) < 1$ if and only if $-192\mu^3 + 96\mu^2 + 112\mu - 129 < 0$. This is true since this is a decreasing function ($-576\mu^2 + 192\mu^2 + 112 < 0$) when $\mu \geq 1$ and $-192 + 96 + 112 - 129 < 0$.

Proof of lemma 3

We will now show that to the right of 1, the function f is either always increasing, or first decreasing and then increasing. Since $f(1) < 0$, and the function has to be eventually positive, the result will follow.

Suppose first that $f'(1) = \frac{k_A}{k_B}(\gamma+1) - 4 \geq 0$. Then, since $\gamma \geq 2$, we have that $\frac{1}{2}f''(1) = \frac{k_A}{k_B}(\gamma+1)\frac{\gamma}{2} - 4 \geq 0$. Since f' is a convex function to the right of 1, it must be true for $\mu \geq 1$ that $f'(\mu) \geq f'(1) + f''(1)(\mu-1) \geq 0$. Now, assume that $f'(1) < 0$. Let μ^* be the lowest value of $\mu \geq 1$ such that $f'(\mu^*) = 0$.

Since $f'(1) < 0$, this implies that $f'(\mu)$ must be increasing at $\mu = \mu^*$. Thus $f''(\mu^*) \geq 0$. Since f' is a convex function to the right of 1. Thus, for $\mu \geq \mu^*$, we have that $f'(\mu) \geq f'(\mu^*) + f''(\mu^*)(\mu - \mu^*) = f''(\mu^*)(\mu - \mu^*) \geq 0$, and $f'(\mu) \geq 0$, for $\mu \geq \mu^*$. Since μ^* is the lowest value of $\mu \geq 1$ such that $f'(\mu^*) = 0$, then $f'(\mu) \leq 0$, for $\frac{7}{4} \leq \mu \leq \mu^*$.

Proof of Proposition 4

We have that $\bar{p} - p_l = \bar{p} - \bar{p}/2\mu$, and $u_h - u_l = (\mu - 1)u_l$. This means that

$$\frac{D_h}{D_l} = \frac{\frac{S}{\theta} \left(\bar{\theta} - \frac{\bar{p}(\frac{2\mu-1}{2\mu})}{(\mu-1)u_l} \right)}{\frac{S}{\theta} \left(\frac{\bar{p}(\frac{2\mu-1}{2\mu})}{(\mu-1)u_l} - \frac{\bar{p}}{u_l} \right)} = \frac{2\bar{\theta}(\mu-1)u_l}{\bar{p}} - \frac{2\mu-1}{2\mu}.$$

If we substitute into this expression the value for u_l we have:

$$\frac{D_h}{D_l} = 2\bar{\theta}(\mu-1)\bar{p}^{-\frac{\gamma-1}{\gamma+1}} \left[\frac{S}{k_B 4\bar{\theta}(\mu-1)^2} \right]^{\frac{1}{\gamma+1}} - \frac{2\mu-1}{2\mu}.$$

Since μ does not depend on \bar{p} , and $\gamma > 1$, this function is clearly decreasing in \bar{p} .

Proof of Lemma 6.

$$\begin{aligned} CS_h &= \frac{Su_h}{2\bar{\theta}}[\bar{\theta}^2 - \theta_{hl}^2] - S\bar{p} \left(1 - \frac{\theta_{hl}}{\bar{\theta}} \right) \\ &= \frac{S\mu u_l}{2\bar{\theta}} \left[\bar{\theta}^2 - \left(\frac{\bar{p}(2\mu-1)}{2\mu(\mu-1)u_l} \right)^2 \right] - S\bar{p} \left[1 - \frac{\bar{p}(2\mu-1)}{\bar{\theta}2\mu(\mu-1)u_l} \right] \\ &= \frac{S\mu u_l \bar{\theta}}{2} - \frac{S}{2\bar{\theta}} \left[\frac{\bar{p}^2(2\mu-1)^2}{4\mu(\mu-1)^2 u_l} \right] - S\bar{p} \left[1 - \frac{\bar{p}(2\mu-1)}{\bar{\theta}2\mu(\mu-1)u_l} \right] \\ &= \frac{S\mu u_l \bar{\theta}}{2} - \frac{S(2\mu-1)\bar{p}^2}{2\bar{\theta}\mu(\mu-1)u_l} \left[\frac{3-2\mu}{4(\mu-1)} \right] - S\bar{p} \\ &= \frac{S\mu u_l \bar{\theta}}{2} - \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{(3-2\mu)(2\mu-1)}{2\mu} \right] - S\bar{p} \end{aligned}$$

$$CS_l = \frac{Su_l}{2\bar{\theta}}[\theta_{hl}^2 - \theta_{l0}^2] - p_l D_l$$

$$\begin{aligned}
&= \frac{Su_l}{2\bar{\theta}} \left[\left(\frac{\bar{p} \left(\frac{2\mu-1}{2\mu} \right)}{(\mu-1)u_l} \right)^2 - \left(\frac{\bar{p}}{2\mu u_l} \right)^2 \right] - \frac{S\bar{p}^2}{4\bar{\theta}(\mu-1)^2 u_l^2} \\
&= \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{(3\mu-2)}{2\mu} \right] - k_B^{\frac{2}{\gamma+1}} \left(\frac{S\bar{p}^2}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma-1}{\gamma+1}}.
\end{aligned}$$

So

$$\begin{aligned}
CS_h + CS_l &= \frac{S\mu\bar{\theta}}{2} \left(\frac{S\bar{p}^2}{k_B 4\bar{\theta}(\mu-1)^2} \right)^{\frac{1}{\gamma+1}} + \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{4\mu^2 - 5\mu + 1}{2\mu} \right] \\
&\quad - k_B^{\frac{2}{\gamma+1}} \left(\frac{S\bar{p}^2}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma-1}{\gamma+1}} - S\bar{p}
\end{aligned}$$

$$\begin{aligned}
\Pi_h &= \frac{S\bar{p}}{\bar{\theta}} \left[\bar{\theta} - \frac{\bar{p}(2\mu-1)}{2\mu(\mu-1)u_l} \right] - k_A \frac{\mu^\gamma u_l^\gamma}{\gamma} \\
&= S\bar{p} - \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{2(\mu-1)(2\mu-1)}{\mu} + \frac{k_A \mu^\gamma}{\gamma} \right].
\end{aligned}$$

$$\begin{aligned}
\Pi_l &= \frac{S\bar{p}^2}{4\bar{\theta}\mu(\mu-1)u_l} - \frac{k_B}{\gamma} u_l^\gamma \\
&= \left(\frac{S\bar{p}^2 k_B^{1/\gamma}}{4\bar{\theta}(\mu-1)^2} \right)^{\frac{\gamma}{\gamma+1}} \left[\frac{\mu-1}{\mu} - \frac{1}{\gamma} \right]
\end{aligned}$$

Proof of Proposition 7

Both SW and $\Pi_h + \Pi_l$ are concave functions. The function $\Pi = S\bar{p} - D\bar{p}^{\frac{2\gamma}{\gamma+1}}$, so that its argmax (as a function of \bar{p}) occurs for $\hat{p} = \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^{\frac{\gamma+1}{\gamma-1}}$.

Also, the function $SW = A\bar{p}^{\frac{2}{\gamma+1}} - B\bar{p}^{\frac{2\gamma}{\gamma+1}} - C\bar{p}^{\frac{2(\gamma-1)}{\gamma+1}}$. Its maximum occurs when $\frac{2}{\gamma+1}A\bar{p}^{\frac{1-\gamma}{\gamma+1}} - \frac{2\gamma}{\gamma+1}B\bar{p}^{\frac{\gamma-1}{\gamma+1}} - \frac{2(\gamma-1)}{\gamma+1}C\bar{p}^{\frac{\gamma-3}{\gamma+1}} = 0$, or, equivalently, when $A - \gamma B\bar{p}^{\frac{2(\gamma-1)}{\gamma+1}} - (\gamma-1)C\bar{p}^{\frac{2(\gamma-2)}{\gamma+1}} = 0$. Denote this maximum value by \hat{p} . Denote by $F(x)$ the following function: $F(x) = A - \gamma Bx^{\frac{2(\gamma-1)}{\gamma+1}} - (\gamma-1)Cx^{\frac{2(\gamma-2)}{\gamma+1}}$. \hat{p} is

defined by $F(\widehat{\hat{p}}) = 0$. Since $F(\cdot)$ is a decreasing function, if $F(\widehat{p}) < 0$, then $\widehat{\hat{p}} < \widehat{p}$.

But $F(\widehat{p}) = A - \gamma B \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^2 - (\gamma - 1)C \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^{\frac{2(\gamma-2)}{\gamma-1}}$, which implies that $F(\widehat{p}) < 0$, if and only if $A < \gamma B \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^2 + (\gamma - 1)C \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^{\frac{2(\gamma-2)}{\gamma-1}}$ and the results follows.

Proof of Corollary 8

If $\gamma \rightarrow \infty$, then $\mu \rightarrow 1$. Also, from the equation that implicitly defines μ one can see that $\mu < \left(\frac{4k_B}{k_A} \right)^{\frac{1}{\gamma-1}}$. This means that $\frac{k_A \mu^{\gamma+1} + \mu}{\gamma} \rightarrow 0$. Which implies that

$$\frac{4(\mu - 1)^2(\gamma + 1) \left[2\mu^2 - \mu + 1 + \frac{k_A \mu^{\gamma+1} + \mu}{\gamma} \right]}{2\gamma \left[(\mu - 1)(4\mu - 3) + \frac{k_A \mu^{\gamma+1} + \mu}{\gamma} \right]^2} \rightarrow \infty$$

Proof of Corollary 9

If $k_A = 0$, we have that $\gamma \frac{B}{A} \left(\frac{S}{\frac{2\gamma}{\gamma+1}D} \right)^2 = \frac{4(\mu-1)^2(\gamma+1)^2 \left[2\mu^2 - \mu + 1 + \frac{\mu}{\gamma} \right]}{2\gamma \left[(\mu-1)(4\mu-3) + \frac{\mu}{\gamma} \right]^2}$. Since $\gamma \geq 2$, we have that

$$\begin{aligned} \frac{4(\mu - 1)^2(\gamma + 1)^2 \left[2\mu^2 - \mu + 1 + \frac{\mu}{\gamma} \right]}{2\gamma \left[(\mu - 1)(4\mu - 3) + \frac{\mu}{\gamma} \right]^2} &> \frac{9(\mu - 1)^2 \left[2\mu^2 - \mu + 1 \right]}{\left[(\mu - 1)(4\mu - 3) + \frac{\mu}{2} \right]^2} \\ &= \frac{18\mu^4 - 45\mu^3 + 9\mu^2 - 27\mu + 9}{16\mu^4 - 52\mu^3 + \frac{265}{4}\mu^2 - 39\mu + 9} \end{aligned}$$

But if $2\mu^3 + 7\mu^2 - 58\mu + 12 > 0$, then $\frac{18\mu^4 - 45\mu^3 + 9\mu^2 - 27\mu + 9}{16\mu^4 - 52\mu^3 + \frac{265}{4}\mu^2 - 39\mu + 9} > 1$ and the result follows.

