

**Title: Modelling the redistribution of hospital supply to achieve equity taking account of patient's behaviour**

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**Abstract:** The pursuit of geographic equity objectives within a NHS-institutional setting requires information on how to change the distribution of supply to achieve greater equity in access and utilisation. Previous methods for analysing the impact of hospital changes have not accounted for competing definitions of equity and have relied on crude assumptions on patients' behaviour in using hospitals. The approach developed in this study is a multi-modelling one based on different mathematical programming location-allocation models (LAMs) to redistribute hospital supply and with single objective functions. We developed: three LAMs that pursue alternative equity objectives (captured by the objective function), make different interpretations of the redistribution problem, and use different assumptions about the utilisation behaviour of patients (captured by constraints); and a framework for the evaluation of outputs from the models (including impacts on utilisation rates, differences between supply and need, distance travelled by patients, total utilisation and scale of redistribution). The underlying structure of each LAM is: 1) minimisation of distance taken to use hospital care, with patients aiming to use the closest hospitals; 2) minimisation of variations in utilisation rates, with patients using hospitals as depicted by past behaviour described by a gravity model; 3) and minimisation of distances to the closest hospitals, with patients using hospitals as depicted by a flow demand model to predict hospital utilisation. The LAMs have been applied on Portuguese data. The pursuit of distinct equity objectives and of certain assumptions of patients' behaviour produces different redistributive results. There were trade-offs when different redistributive results were evaluated under different policy objectives (such as between equity of geographic access and of utilisation or between equity of geographic access and total utilisation). Results indicate a pattern of redistribution for Portugal favouring hospitals in the interior, the south, and peripheral hospitals in urban areas.

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## **1 Objectives**

The pursuit of geographic equity objectives within a NHS-institutional setting requires information on how to change the distribution of supply to achieve greater equity in access. Previous methods for analysing the impact of hospital changes have focused on the distribution of finance (and not how to redistribute hospital capacity), not accounted for competing definitions of equity, have relied on crude assumptions on patients' behaviour in using hospitals, and have been weak in modelling interaction between the use of alternative hospitals and for the process of demand for hospital care. This paper develops models that test how redistribution of hospital supply can be used to target improvements in geographical equity, for three formulations of the concept of equity of access and utilisation. Those three formulations correspond to three alternative models that not only relate to different concepts of equity but also test different assumptions about patients' behaviour and illustrate the impacts of different assumptions on methodological choices (as a type of sensitivity analysis).

This paper reports the development of generalisable models to redistribute hospital supply, with application of the models illustrated for a country with a National Health Service (NHS), namely Portugal which is similar to the countries of the UK and Italy in being committed to equity (Rice and Smith 1999)<sup>2</sup>. The models attempt to account for the characteristics of NHS type health care systems that seem to be resulting in significant variation in access and utilisation (Oliveira 2003). The starting point of this study is a review of the related literature to redistributive models of hospital supply. The following section presents the experimental design used in building redistributive models and sets an analytical framework for the analysis of outputs, so as to compare implications of alternative models. The next section proposes a set of alternative models to analyse hospital redistribution, addressing the weaknesses of previous models. The final section presents and discusses results.

## **2 Literature review**

This review describes location-modelling literature and frames the choice of a modelling approach. It is structured in three sub-sections which discuss: methodological issues in the literature on location, multi-spatial objectives and available models, and specific issues in the

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<sup>2</sup> For example, as Table 6 below shows, for Portugal, some areas use hospital resources at a rate five times higher than the national rate while other areas use hospital resources at one third of the national rate.

application of models in the area of health care. We focus in the literature that is more relevant to the public and health care sectors.

In building new models we aim at overcoming a number of weaknesses of earlier methods applied to the health sector. First, in using realistic assumptions about the criteria of patients' choice of hospital so that models can be used to predict changes in utilisation. Second, in accounting for interaction between hospital supply and utilisation of alternative hospitals that is vital for prediction purposes. Although gravity models deal with interaction, they do so in an unsatisfactory way<sup>3</sup>. Third, in accounting for the characteristics of the health care system in estimating demand for hospital care, or of how the process of hospital demand is formulated. Thus, we propose a different set of models that give some insight into the problem of how redistribution of hospital supply might be proposed. Thus, our literature review focuses on these aspects.

### 2.1 Methodological issues for location literature

Two different modelling approaches have been used to optimise the geographical distribution of capacity in geographic and operational research literature. *Location models* assume that consumers' responses are made independently of provider conditions (Rushton 1987). *Location-allocation models* simultaneously allocate supply and consumers demand (Hodgson, Rosing, and Storrier 1996). Table 1 shows how these approaches compare in modelling consumer behaviour (Love, Morris, and Wesolowsky 1988). The approach of location-allocation models (LAMs) is obviously better for modelling hospital capacity to equalise access to hospital care.

**Table 1: Location and location-allocation models vs. variables to be calculated within the models**

<i>Models</i>	<i>Individual hospital supply</i>	<i>Individual demand</i>	<i>Total demand</i>
Location	Calculated within model	Independent of supply	Fixed total demand
Location-allocation	Calculated within model	Dependent on supply	Calculated within model

Three main methods have been used for analysing geographic distribution and redistribution of public facilities (including hospital supply): *spatial interaction models* (SIMs), *entropy models* (EMs) and

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<sup>3</sup> Traditional gravity models respect the property of independence of irrelevant axioms, in that the flows to any destination are independent of other destinations. A recent study from Congdon (Congdon 2001) has improved the interaction mechanism of gravity models in the context of modelling emergency flows. Congdon has adapted a gravity model that is more responsive to changes in the patterns of supply, using Bayesian methods to re-estimate some of the parameters of the gravity model (in order to represent new accessibility scores given supply changes), and afterwards re-running the gravity model with the new parameters. This approach, however, requires local knowledge to specify the new parameters for changes in supply and is thus difficult to use for other than small local studies.

*mathematical programming models* (MPs)<sup>4</sup>. These models represent variations in the elements that characterise a LAM (for example, in decision criteria)<sup>5</sup>; use distinct types of information on spatial behaviour; and have different potentials for describing or predicting behaviour. A brief description of the strengths and weaknesses of each group of models follows. Nonetheless, specific versions of the MP, EM and SIM can be applied and have previously been shown to produce similar results, mainly when the models are used to replicate behaviour<sup>6</sup>.

#### 2.1.1 Spatial interaction models

SIMs are a form of probability interaction modelling (O'Kelly 1987), and use information on population numbers, hospital size, distance and a decay function to reproduce flows of consumers (or patients). SIMs can replicate the current pattern of patient flows between demand and treatment zones (for example (Mayhew, Gibberd, and Hall 1986) (Taket 1989) (Cho 1998)). However, they are inadequate in predicting user flows in response to supply changes because they assume that when there are changes in one hospital all the other hospitals gain in comparison with their shares of utilisation prior to that change (McLafferty 1988) (Porell and Adams 1995). Secondly, SIMs do not consider neither local variations from the local health system, nor the hierarchical and organisational structure of hospitals. SIMs cope with methodological problems (McLafferty 1988), such as SIMs assume that populations use hospitals in accordance with a decay parameter and this implies assuming that both urban and rural populations are able to use both urban and rural hospitals and this might not be verified in practice<sup>7</sup>. However, there might be a scope for improving SIM models. For example by developing unconstrained gravity models which account for local variations.

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<sup>4</sup> Evaluation and appraisal techniques such as cost benefit analysis, cost effectiveness analysis and cost utility analysis models are seen as not appropriate for comparing alternative improvements to the hospital network (Oliveira 2003). This is because these techniques would imply comparison of multiple scenarios of redistribution, and would not provide a simple tool for analysing changes in a network of hospitals.

<sup>5</sup> To adequately solve a location-allocation problem a model with four elements is required (Ghosh and Rushton 1987): decision criteria on the objective or objectives to be attained; rules for consumer behaviour with respect to the spatial choice of hospital; a representation of the environment, such as the choice of the geographic level of analysis, with the need to determine travel costs, times or distances; and a choice between deterministic model and stochastic model. Most of these analytical choices entail judgements (Mandell 1991).

<sup>6</sup> In the context of description of flows, these models can also be translated into random utility models. For example, the SIM/gravity model may be applied from random utility theory (characterised by rational choice behaviour) (Williams and Senior 1978); a maximum entropy formulation can be a starting point for building a logit/utility theory model (Jornsten and Lundgren 1989); when the cost exponent in a gravity model tends to infinity (that is, the elasticity of utilisation to distance in the decay function is infinite), the total distance travelled tends towards a minimum, and the trip distribution tends to a linear programming assignment with patients travelling to the closest point (O'Kelly 1987).

<sup>7</sup> Analysis of the database used in the empirical application shows that almost 100% of population of urban areas are attended in urban hospitals, and thus do not seem to be very willing to use rural hospitals.

### 2.1.2 Entropy models

EMs have been used in different contexts and disciplines (such as thermodynamics, statistical dynamics, statistics and information theory (Wu 1997) (Fang, Kajasekera, and Tsao 1997) (Arndt 2001)) and are a type of mathematical programming model that makes use of the first principle of data reduction (Wu 1997): when there are incomplete data, the solution must include and be consistent with all relevant available data. Mainstream EMs replicate the macro-properties of the system with information on users, on levels of supply and on accessibility costs among users. EMs emphasise short-term prediction (given fixed supply) (Webber 1978) and are adequate when there is a lack of theoretical understanding of what to include in the model (Anas 1983) (this might be particularly relevant for developing countries). Also, they avoid the assumption of micro-economic models based on utility theory, may be derived from a theoretical set-up and examine only small components of the decisions of individuals (Webber 1978). The weaknesses of EMs lie in that they follow a holistic view, which imposes system constraints that do not consider the ways in which groups make spatial choices (Nijkamp 1978) –i.e., they do not account for the process of health care demand, in particular for the determinants of individuals' decisions. Also EMs depend highly on the formulation of consumers' travelling costs.

### 2.1.3 Mathematical programming models

MPs have been widely used for locating and allocating public facilities. They maximise an equity objective (for example, distance travelled by users), assume some type of user behaviour (for example, patients travel to the closest hospital), and use constraints that reflect characteristics of the health care system. Table 2 presents (and explains) a number of choices of MP models based on some technical variants that are relevant for the public sector: the choice between a single or a multiple objective function; optimal or non-optimal solution models; and nested or hierarchical models. Multiple objective MP models might be more realistic but entail several judgements on weighting the criteria and applying complex algorithms (Martin et al. 2000) (Gonzalez 2001). Heuristic models are required when the algorithms built are not efficient or unable to compute an optimal solution in reasonable time. Hierarchical models capture variations in the facilities network, deploy a complex formulation and often require heuristic models to arrive at a solution (heuristic models are defined in Table 2).

**Table 2: Variety of MP models for public and health care facilities location**

<i>MP models under several classifications</i>	<i>Description</i>	<i>Examples from literature</i>
Single- vs. multiple objective models	<i>Single objective models</i> minimise a single objective, such as: distance from users to the closest provider point, or maximum distance; or total travelling costs.	(Mohan 1983)
	<i>Multiple-objective models</i> consider several objectives; examples of multiple objectives models are goal programming, multi-criteria utility models or bi-criteria models.	(Rushton 1987)

Optimising vs. heuristic models	<i>Optimising models</i> provide an optimal solution.	(Mohan 1983)
	<i>Heuristic models</i> produce satisfactory results that may not be an optimal solution, but a second best solution (given technical difficulties in solving these models).	(Bennet 1981)
Nested vs. hierarchical models	<i>Nested models</i> represent facilities at different hierarchical levels and treat all the facilities of the hierarchy as if they were at the same level (i.e. as if no hierarchy applied) assuming that all the facilities provide the same set of services.	(references in (Marianov and Serra 2001))
	<i>Hierarchical models</i> represent a referral system, such as top-down and bottom-up models of referral formulation; and might account for total travel costs incurred in accessing different levels of facilities.	(Church and Eaton 1987) (Rahman and Smith 2000)

The strengths of the MP approach are flexibility in the choice of objective function and constraints on the system, and their ability to provide a global solution. Weaknesses of MP models have included the use of crude assumptions about users' behaviour, such as users travelling to the closest facility (Current, Min, and Schilling 1990) (although we believe that this assumption can be changed); and the process of building MP models has been highly restricted by the assumption of linearity (although MP is still capable of producing efficient approximation models). As the key objective of this paper is to use LAMs to focus on behavioural patterns of patients and on goals to be achieved by policy-makers, we have chosen the MP approach as it is flexible to deal with these objectives.

## 2.2 Specific issues in health research

Some LAMs have been applied to the health sector, and their objectives and context of application have influenced the structures of the models and the selection of techniques. The choice of model depends upon prior beliefs with regard to *demand for health care function, the equity objectives to be pursued and the constraints that characterise the health care system*. A wider review on the literature on this is available on (Oliveira 2003). No models are suitable to deal with our full set of objectives of dealing with the redistribution of current capacity, with the process of demand of hospital care (and patients' behaviour), and with alternative objectives to be followed by policy-makers. So we purpose the set of model that follow.

## 3 Experimental design

The approach developed in this paper is a *multi-modelling one* based on *different mathematical programming models with single objective functions*. The MP approach is flexible in the choice of objective function, of constraints of the health care system and of the limits to redistribution. A multi-modelling approach corresponds to the use of a set of models described in Table 3 for analysing the redistribution problem. The single objective function is based on equity improvements in only one dimension, as multiple-objective functions create problems in the

solution of the algorithms. Rather than to try to incorporate multiple objectives within a single objective function, here produced models have different objective functions.

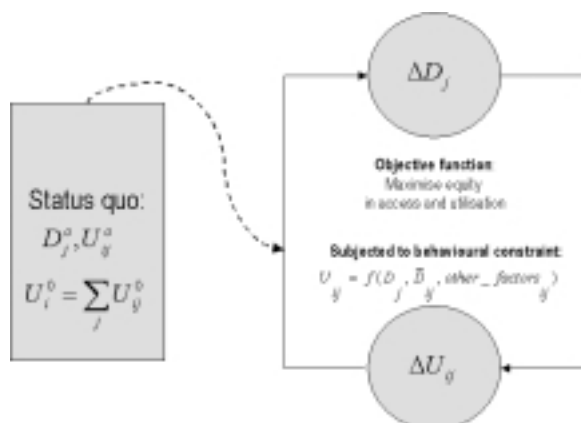
The three models correspond to different conceptualisations of the health care system (for example on the connection between hospital and primary care), have distinct (and alternative) equity objectives, make different interpretations of the redistribution problem, use different assumptions about the utilisation behaviour of patients with regard to different levels of hospital supply, and use different constraints on redistribution (such as using a lower bound for change in individual hospital capacity). Nonetheless, any single objective function tends by its very nature to be incomplete (Rushton 1987) as policy makers pursue several objectives. In order to account for these objectives, we analyse the redistributive results of the three models on a set of policy objectives, which indicates the degree of similarity of results using different assumptions, and may be seen as an exercise in sensitivity analysis.

This section describes the generic structure of the three models and the reasoning for the use of a mixed modelling strategy, as well as introduces a framework for analysis and comparison of outputs of the models.

### 3.1 Generic structure

Figure 1 presents the general structure of the MP models, organised according to the following principles. The starting point is information on the *status quo* with regard to location of hospital facilities and on the current patterns of utilisation, as the models aim to redistribute hospital supply ( $D_j^0$ ,  $U_{ij}^0$  and  $U_i^0$  stand for current level of supply of hospital  $j$ , past flows between population  $i$  and hospital  $j$ , and (past) total level of utilisation of population  $i$ , respectively). The objective is to improve equity in the utilisation or access of public hospitals by the redistribution of public hospital supply ( $D_j$ s). The  $D_j$ s are the decision variables. Changes in the  $D_j$ s imply new levels of utilisation flows ( $U_{ij}$ s). The distribution of new levels of  $U_{ij}$  must be analysed according to a set of desirable criteria. These criteria are synthesised below in the framework for analysis of outputs. The underlying behaviour of patients is made explicit in the three models, with behaviour represented in different constraints and structure of the MP models. Each model aims to capture the process of demand for hospital care in a different way, given current system characteristics.

**Figure 1: Simultaneous spatial redistribution of hospital supply and variations of utilisation flows in a location-allocation model**



The development and choice of models was guided by the need to find algorithms that produce efficient solutions. This is a problem both because of the computational burden imposed by the large number of potential locations and population points (68 and 275, respectively, in Portugal) and the difficulty of solving certain types of models (for example, MP models with integer variables).

The three models were chosen because they present different strengths: the distance-based model (DBM) has a simple structure that generates changes in supply in order to improve location accessibility; the utilisation-based model (UBM) uses an objective function that is clearly consistent with objectives of health policy (equalisation of utilisation rates); and the utilisation flows-based model (UFBM) accounts for the most realistic information on patients' use of hospitals following changes in supply. The distance-based and utilisation-based models (DBM and UBM) are based on previous models that have been described above and have been adapted to meet the specific aims of this paper. The DBM changes the assumption that patients make use of a single hospital and hence is an improvement on the nearest centre model; the UBM makes use of information on patient travelling behaviour taken from an attraction-constrained gravity model, and differs to previous SIMs in that it minimises variations in utilisation rates by population area. The UFBM takes a quite different approach to existing models. It focuses on realistic assumptions about patients' utilisation behaviour in response to changes in hospital supply levels (it uses the equations of the flow demand model (FDM) (Oliveira 2002) as a constraint for predicting hospital utilisation); and uses a distinct objective function based on a principle of location accessibility.

The three models deployed share a set of features for the modelling of redistribution. They structure a multi-hospital system with hospital facilities 'competing' for limited capacity fixed at

the current level. They follow a second best approach as they use a predetermined and finite set of potential locations based on current hospital sites. Hence, may be described as optimisation models (optimising an equity objective) or as simulation models (testing the consequences of changes in hospital supply on the objective function). The three models can be compared across four characteristics, structured in four columns in Table 3:

1. *Whether they are location or allocation models,*
2. *Choice of equity index in the objective function.*
3. *Use of normative vs. prescriptive assumptions for patient behaviour.*
4. *Use of constraints that impose the redistributive nature of the model and choice of a mechanism for capturing the interaction* between hospital supply and utilisation levels of alternative hospitals. For example, limits to levels of redistribution (per hospital) only apply to some models.

### **3.2 Framework for analysis of outputs**

Figure 2 structures the set of alternative objectives to be achieved in a value tree, which is a hierarchical representation of objectives structured from general and abstract ones (at the top) to measurable and well-specified criteria (at the bottom) (Goodwin and Wright 1998). Three main objectives are pursued, which correspond to the top objectives of the value tree (the corresponding measurement indicators are described in Table 4):

- a) *Maximisation of equity improvements.* This objective can be divided into three more specific goals:
  - *Maximum equity of utilisation.* This means minimising variations of utilisation rates between population areas.

**Table 3: Qualitative description of MP models**

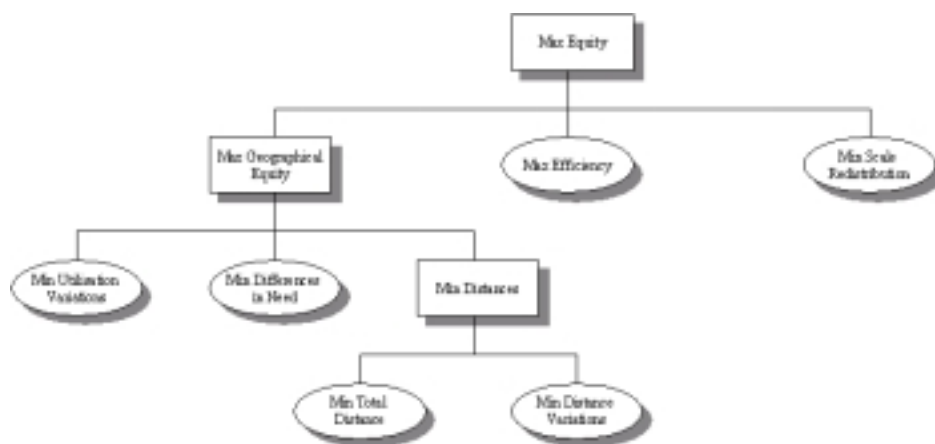
<i>Model (and typology)</i>	<i>Key equity concept</i>	<i>Patients' behaviour constraint</i>	<i>Supply interaction and other system constraints</i>
DBM -Distance-based model Location model	Equity of access: minimisation of total distances travelled by patients to (closest) hospitals.	Assumption that patients travel to closest hospitals, and patients are allocated to three types of hospitals in accordance with past quotas (normative).	No interaction between hospital supply and utilisation of alternative hospitals. Patients make use of three hospitals: one central hospital and two other hospitals of the network.
UBM -Utilisation-based model Location-allocation model	Equity of utilisation by population area: minimisation of sum of variations between predicted and normative utilisation per population area (according to need).	Assumption that patients in each geographic area use hospitals based on fixed conditional probabilities (prescriptive).	No interaction between hospital supply and utilisation of alternative hospitals. System constraints: upper and lower limits to variation in the supply of each hospital; fixed total supply. Conditional probability of use of hospital generated by a gravity model.
UFBM -Utilisation flows-based model Location-allocation model	Equity of utilisation flows between population areas and hospitals with the equity target defined as: flows if patients were treated in the closest hospital.	Assumption that patients in each geographic area use hospitals in accordance with the FDM developed in (Oliveira 2002) (prescriptive).	Interaction constraint as captured by the FDM constraint. System constraints: upper and lower limits to variations in the supply of each hospital; fixed total supply. Fixed probabilities of a population making use of a hospital (prediction taken from the first part of the FDM)

- *Minimisation of differences between supply and need at district level.* Since equal opportunity of access for those in equal need across geographic areas is also a policy objective, it is desirable that redistribution of supply in the MP models minimises differences between supply and need at the district level<sup>8</sup>.
  - *Minimisation of distance travelled.* Shorter travel distance to access hospital care implies lower costs for patients to access health services. Both total distance and distance travelled per population area are important in this context.
- b) *Maximisation of efficiency.* Efficiency is specifically defined here as the total level of (potential or predicted) production in the system, as measured by utilisation. Utilisation of hospital declines with the distance because patients have to travel –but this varies by hospital type, and flows depend on hospital size (Oliveira 2002). Hence, redistribution of supply will impact on patient flows and total utilisation (for the UBM and UFBM). The higher the total utilisation, the better.
- c) *Minimisation of the scale of redistribution in terms of ‘winners’ and ‘losers’* from changes in supply.

**Table 4: Measures of equity and policy related criteria**

Criteria	Measure
Equity- utilisation	Utilisation rates by small area
Equity –supply/need	Variations in the levels of supply at the district level in comparison to needs-based estimates
Equity -distance	Average distance to be travelled by population area Total average distance in the system
Efficiency	Total utilisation in the system
Redistribution	Numbers of ‘winners’ and ‘losers’ as a result of redistribution at the hospital level

**Figure 2: Value tree representing equity and policy-related criteria**



<sup>8</sup> Hence, the outputs from each model are juxtaposed with the estimated distribution of needs for the Portuguese district level, presented in (Oliveira and Bevan 2003).

## **4 Models for improving geographic equity**

This section describes each model in turn giving an intuitive account, a description of the structure of the MP program, and the formulation of the model. Table 5 gives a summary of the key features of the quantitative structure of the three models.

### **4.1 Distance-based model -DBM**

This model uses information from the utilisation matrix (described in (Oliveira 2002)), in which each population area makes use of a small number of hospital sites, and all population areas use central hospital sites (which are the only providers for highly specialised services). The DBM aims at answering the following question: which supply changes would be required for a higher proportion of demand to be met by the closest hospitals?

The DBM follows the classical structure of the 'p-median' model that minimises distance weighted populations. The classic p-median model assumes that patients are treated in one hospital only but this is unrealistic and incompatible with characteristics of hospital systems, with diversity in supply, differences in attractiveness and inelastic demand characteristics that might justify the use of more than one hospital (O'Kelly 1987). In the DBM, users are allocated to three hospitals –the closest, second closest and closest central hospitals-, under previous fixed quotas (or weighted populations, as explained below). The quota of patients allocated to the closest hospital is increased in comparison to past quotas, which implies that this model represents improvements in current accessibility as supply from other hospitals is transferred to the closest hospitals. The objective function of the DBM minimises distances travelled by patients to access the three hospitals used, and this formulation is deployed because empirical evidence contradicts the rule that patients tend to use the closest hospital.

The model produces: new supply levels and new distances to be travelled by patients as outputs. It is assumed that utilisation of hospital services (and population demand) for some hospitals is fixed.

**Table 5: Quantitative differences between models (but key features)**

Objective function	Behavioural constraint	Other constraints
<p><math>\min \sum_i \left[ U_{-1_i} * d_{-1_i} + U_{-2_i} * d_{-2_i} + U_{-c_i} * d_{-c_i} \right]</math> (equity in location accessibility)                      Decision variables: <math>d_{-1_i}, d_{-2_i}, d_{-c_i}, D_j</math>                      Where, for example: <math>d_{-1_i} = \sum_j Flow_{-1_{ij}} * d_{ij}</math></p>	<p><math>Flow_{-1_{ij}} = \begin{cases} 1: \text{if } -j\_first\_hospital\_use \\ 0: \text{otherwise} \end{cases}</math>  <math>Flow_{-2_{ij}} = \begin{cases} 1: \text{if } -j\_second\_hospital\_use \\ 0: \text{otherwise} \end{cases}</math>  <math>Flow_{-c_{ij}} = \begin{cases} 1: \text{if } -j\_central\_hospital\_use \\ 0: \text{otherwise} \end{cases}</math>                      Fixed: <math>U^0 = \sum_i U_i</math></p>	<p>Fixed utilisation per area in proportion to national utilisation rate:  <math>U_i = \bar{U}^N * W_i</math>                      Utilisation per hospital type (deterministic):  <math>U_{-1_i} = share_{-1_i} * U_i,</math> <math>U_{-2_i} = share_{-2_i} * U_i,</math>  <math>U_{-c_i} = share_{-c_i} * U_i</math></p>
<p><math>\min \sum_i  U_i - U_i^N </math> (equity of access per population area)                      Decision variables: <math>D_j, U_{ij}</math></p>	<p><math>U_{ij} = \tilde{p}_{ij} * D_j</math>  <math>U_i</math> and <math>U</math> are unconstrained.</p>	<p>Upper and lower limits in variation of hospital supply:  <math>\min\_D_j \leq D_j \leq \max\_D_j</math>                      Fixed total supply: <math>\sum_j D_j = D</math>. Constraint defining total utilisation:  <math>U_i = \sum_j U_{ij}</math>. Normative utilisation: <math>U_i^N = W_i * U^0 / W</math></p>
<p><math>\min \sum_i \sum_j (\log U_{ij} - \log U_{ij}^r)^2</math> (equity of utilisation)                      Decision variables:  <math>D_j, \log U_{ij}'</math></p>	<p><math>\log U_{ij} = \log \hat{p}_{ij} + \log U_{ij}'</math> and  <math>\log U_{ij}' = \text{other}_{ij} + \hat{\alpha}_0 * D_j + \hat{\alpha}_1 * \tilde{D}_{ij} + \hat{\alpha}_2 * DumFirst_{ij} * D_j</math>  <math>+ \hat{\alpha}_3 * DumSecond_{ij} * D_j + \hat{\alpha}_4 * DumCentral_{ij}</math>  <math>+ \hat{\alpha}_5 * DumPorto_{ij} * D_j + \hat{\alpha}_6 * DumCoimbra_{ij} * D_j</math>  <math>+ \hat{\alpha}_7 * DumLisboa_{ij} * D_j</math>  <math>U_i</math> and <math>U</math> are unconstrained.</p>	<p>Upper and lower limits for variation of hospital supply:  <math>\min\_D_j \leq D_j \leq \max\_D_j</math>                      Fixed total supply: <math>\sum_j D_j = D</math>. The <math>\alpha</math>'s and <math>\log \hat{p}_{ij}</math> are parameters estimated in the FDM.</p>

#### 4.1.1 Structure of the program

*Objective function.* The model minimises the distances to be travelled by patients to reach the three hospitals used, weighted by utilisation numbers. This objective function respects the principle of transfers<sup>9</sup> in terms of distance. Utilisation flows depend on needs-weighted population and on past utilisation quotas. The DBM is a linear mixed integer-programming model, as some decision variables are dummies (hospitals of use), while others are continuous (corresponding distances).

*Constraints.* The population uses hospital services at the national utilisation rate. Utilisation per population area is divided into fixed shares for the three types of hospitals (computed from empirical data):

1. At a national level, 10% of area discharges/utilisation on average are served by central hospitals (computed from the database described in (Oliveira 2002));
2. The discharges allocated to the second closest hospital are those observed in 1999 (for example, 15% of patients from a specific population used the second closest hospital);
3. The remaining discharges are allocated to the closest hospital for any population area (this quota is the result of subtracting the quota of patients of the second closest and of the closest central hospital from 100%). This implies a marginal redistribution of capacity towards the hospital point closest to the population.

#### 4.1.2 Formulation

The model takes as *decision variables* the following:  $Flow_{1_{ij}}$ : dummy variable for whether population  $i$  is served by hospital  $j$ , as a first hospital (0,1 values);  $Flow_{2_{ij}}$ : dummy variable for whether population  $i$  is served by hospital  $j$ , as a second hospital (0,1 values); and  $Flow_{c_{ij}}$ : dummy variable for whether population  $i$  is served by hospital  $j$ , as the central hospital (0,1 values);  $d_{1_i}$ : distance travelled between population point  $i$  and the first hospital (non-negative);  $d_{2_i}$ : distance travelled between population point  $i$  and the second hospital (non-negative); and  $d_{c_i}$ : distance travelled between population point  $i$  and the closest central hospital (non-negative).

And as *parameters*:  $d_{ij}$ : Euclidean distance between population point  $i$  and hospital site  $j$ ;  $W_i$ : needs-weighted population for population point  $i$ ;  $share_{1_i}$ : share of population  $i$  that is assumed to go to the first hospital;  $share_{2_i}$ : share of population  $i$  that is assumed to go to the

second hospital; and  $share\_3_i$ : share of population that is assumed to go to the closest central hospital;  $U\_1_i$ : utilisation numbers for population  $i$  and its closest hospital;  $U\_2_i$ : utilisation numbers for population  $i$  and its second closest hospital; and  $U\_c_i$ : utilisation numbers for population  $i$  and its closest central hospital;  $U^0$ : current (past) level of utilisation, on aggregate (*status quo*); and  $\bar{U}^N$ : national utilisation rate.

The model minimises total distance-weighted utilisation, as set in Eq. (I.1).

$$\min \sum_i [U\_1_i * d\_1_i + U\_2_i * d\_2_i + U\_c_i * d\_c_i] \quad (I.1)$$

The model is subjected to fifteen sets of constraints summarised in eight categories. First, total hospital utilisation by a population is assumed to be at the national utilisation rate (Eq.s (I.2) and (I.3)).

$$\bar{U}^N = U^0 / W \quad (I.2)$$

$$U_i^N = W_i * \bar{U}^N, \forall i \quad (I.3)$$

Second, utilisation/demand from a population to each of the three hospitals –closest, second closest and closest central– are fixed (Eq.s (I.4), (I.5) and (I.6)):

$$U\_1_i = U_i^N * share\_1_i, \forall i \quad (I.4)$$

$$U\_2_i = U_i^N * share\_2_i, \forall i \quad (I.5)$$

$$U\_c_i = U_i^N * share\_c_i, \forall i \quad (I.6)$$

Third, each population point has to use three hospitals:

$$\sum_j [Flow\_1_{ij} + Flow\_2_{ij} + Flow\_c_{ij}] = 3, \forall i \quad (I.7)$$

Fourth, for each area, the second hospital has to be different from the first:

$$Flow\_1_{ij} + Flow\_2_{ij} \leq 1, \forall i, j \quad (I.8)$$

Fifth, each population point can only have access to one first, one second and one central hospital (Eq.s (I.8), (I.9) and (I.10)).

$$\sum_j Flow\_1_{ij} = 1, \forall i \quad (I.9)$$

$$\sum_j Flow\_2_{ij} = 1, \forall i \quad (I.10)$$

$$\sum_j Flow\_c_{ij} = 1, \forall i \quad (I.11)$$

Sixth, central hospitals have limited locations:

$$Flow\_c_{ij} = 0, \forall i, \text{ if } j \text{ is not a central hospital} \quad (I.12)$$

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<sup>9</sup> The role of the principle of transfers in the literature on equity indices is the following: the measure of inequity decreases when there are transfers from the best to the worst-off groups.

Seventh, hospital supply (measured by discharges) is defined as the total utilisation by the populations allocated to it, assuming that utilisation by population area is proportional to the national utilisation rate.

$$D_j = \sum_i [Flow_{-1_{ij}} * U_{-1_i} + Flow_{-2_{ij}} * U_{-2_i} + Flow_{-c_{ij}} * U_{-c_i}] \forall j \quad (I.13)$$

Lastly, distances travelled by patients to access the closest, second closest and closest central hospitals are computed as expressed in Eq.s (I.14), (I.15) and (I.16).

$$d_{-1_i} = \sum_j Flow_{-1_{ij}} * d_{ij}, \forall i \quad (I.14)$$

$$d_{-2_i} = \sum_j Flow_{-2_{ij}} * d_{ij}, \forall i \quad (I.15)$$

$$d_{-c_i} = \sum_j Flow_{-c_{ij}} * d_{ij}, \forall i \quad (I.16)$$

#### 4.2 Utilisation-based model -UBM

This model uses an objective function that corresponds to a common health policy objective, namely the minimisation of differences between predicted and normative utilisation per population area. Predicted utilisation is utilisation predicted by a behavioural equation that relates supply ( $D_j$ ) to utilisation flows ( $U_{ij}$ ), as expressed in Eq. (II.1) ( $\tilde{p}_{ij}$  is a conditional probability of population in  $i$  making use of hospital  $j$ ). Normative utilisation is the desirable level of utilisation per population area, if population used resources at the national average utilisation rate.

$$U_{ij} = \tilde{p}_{ij} * D_j, \forall i, j \quad (II.1)$$

Probabilities ( $\tilde{p}_{ij}$ ) are obtained from the application of the gravity model described below. The key problem of using the gravity model to make predictions is the use of the simplistic assumption that when there are changes in the size of one hospital, all the other hospitals gain in proportion to their shares of utilisation prior to that change. The UBM assumes that patients' behaviour is fixed, as described by the (attraction-constrained) gravity model that was applied to Portuguese data. Though the use of a gravity model is inadequate to predict radical changes, as the UBM is a model for only marginal redistribution, the probabilities of flows of populations to hospitals of the gravity model are used as a possible method to predict patients' 'travelling' behaviour. The set of conditional probabilities ( $\tilde{p}_{ij}$ s) is one of the outputs of the gravity model and represents a measure of the propensity of a population to make use of one hospital. The behavioural constraint II.1 uses the assumptions of a destination-varying model<sup>10</sup>, and the  $\tilde{p}_{ij}$  should be interpreted as the probability that a randomly chosen patient living in zone  $i$  is treated

in hospital  $j$  (Wilson and Gibberd 1990). That assumption is an improvement on the DBM, as the UBM assumes that utilisation depends on supply and is thus a LAM. As this is a model for redistribution, a set of upper and lower limits for the maximum allowed variation in supply per hospital are imposed.

#### 4.2.1 Structure of the problem

*Objective function.* The model minimises differences between predicted and normative utilisation numbers. Mayhew and Leonardi (Mayhew and Leonardi 1982) have used a similar model with an objective function that uses the square of the difference between observed/predicted and expected/normative utilisation. The absolute difference between predicted and normative utilisation per population area is preferred, as a utilisation unit has the same value across geographic areas. This objective function respects the principle of transfers with regard to utilisation numbers.

*Constraints.* The probabilities ( $\tilde{p}_{ij}$ ) are outputs of the gravity model and fixed in the behavioural constraint defined in Eq. (II.1). The formula generating the probabilities is defined in Eq. (II.2), which corresponds to the version of the gravity model presented in Eq. (II.3). Since  $\tilde{p}_{ij}$ s are conditional probabilities, their sum is one as in Eq. (II.4).

$$\tilde{p}_{ij} = W_i * f(\beta_j, d_{ij}) * \left[ \sum_i W_i f(\beta_j, d_{ij}) \right]^{-1}, \forall i, j \quad (\text{II.2})$$

$$U_{ij} = \left[ W_i * f(\beta_j, d_{ij}) * \left[ \sum_i W_i f(\beta_j, d_{ij}) \right]^{-1} \right] * D_j, \forall i, j \quad (\text{II.3})$$

$$\sum_j \tilde{p}_{ij} = 1, \forall i \quad (\text{II.4})$$

Lower and upper limits are set for reductions and increases in hospital capacities. As Eq. (II.1) has a limited ability to predict wider changes in supply, upper and lower limits may also be seen as a mechanism of control for avoiding inconsistencies in prediction values (Hallefjord and Jornsten 1984). In addition, these constraints are consistent with the behaviour of countries that will attempt to correct geographical inequities in the redistribution of hospital supply, and that already have a network of sufficient size.

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<sup>10</sup> Under the destination-varying hypothesis, utilisation flows depend on the prior probabilities from the utilisation matrix of the gravity model, and the constant of proportionality depends upon the capacity of the destination (i.e. hospital capacity) (Wilson and Gibberd 1990).

#### 4.2.2 Formulation

The model takes as *decision variables* the following:  $U_i$ : utilisation level of population in site  $i$  (non-negative);  $U_{ij}$ : discharges of population  $i$ , attended in hospital  $j$  (non-negative); and  $U$ : total utilisation (non-negative);  $U_i^N$ : normative utilisation for population site  $i$  (non-negative);  $D_j$ : hospital discharges from hospital site  $j$  (non-negative);  $a_i$ : auxiliary and definitional variable used to obtain an absolute value of differences between utilisation and expected utilisation, per population area  $i$  (non-negative).

And as parameters:  $D^0$ : total supply within the current system, given by  $D^0 = \sum_j D_j^0$ ;  $W_i$ : needs-weighted population in site  $i$  ( $W = \sum_i W_i$ );  $\tilde{p}_{ij}$ : probability of a patient from site  $i$  making use of hospital  $j$ ;  $f_{\min}$ : minimum acceptable percentage of variation in capacity per hospital that is to be maintained; and  $f_{\max}$ : maximum acceptable percentage of variation in capacity per hospital, as a function of current supply;  $\min\_D_j$ : lower limit of the size of hospital  $j$ , computed under the following relationship:  $\min\_D_j = f_{\min} * D_j^0$ ; and  $\max\_D_j$ : upper limit of the size of hospital  $j$ , computed under the following relationship:  $\max\_D_j = f_{\max} * D_j^0$ .

The model minimises absolute differences between predicted utilisation and normative utilisation per population  $i$ , as expressed in Eq. (II.5), following the definitions in Eq.s (II.5a) and (II.5b). The responsiveness of utilisation flows to changes in hospital supply is predicted by the behavioural constraint presented in Eq. (II.1). Normative utilisation ( $\bar{U}^N$ ) is the utilisation level for the geographic area that is proportional to the national utilisation rate computed within the model.

$$\min \sum_i a_i, \quad a_i \geq 0 \quad (\text{II.5})$$

$$a_i = |U_i - U_i^N|, \quad \forall i \quad (\text{II.5a})$$

$$U_i^N = W_i * \bar{U}^N, \quad \forall i \quad (\text{II.5b})$$

The model imposes seven constraints (grouped in five categories). First, a constraint capturing the behaviour of patient flows for different sets of supply distribution (Eq. (II.1)).

Second, two constraints aggregating flows of utilisation per population area and total utilisation (Eq.s (II.6a) and (II.6b)).

$$U_i = \sum_j U_{ij}, \quad \forall i \quad (\text{II.6a})$$

$$U = \sum_i U_i \quad (\text{II.6b})$$

Third, two constraints for setting of upper and lower limits on changes in supply (Eq.s (II.7) and (II.8)).

$$D_j \geq \min\_D_j, \forall j \quad (\text{II.7})$$

$$D_j \leq \max\_D_j, \forall j \quad (\text{II.8})$$

Fourth, a constraint maintaining the current total level of supply in the system (Eq. (II.9)).

$$\sum_j D_j = D^0 \quad (\text{II.9})$$

Fifth, two mathematical constraints for defining the objective function as an absolute value. These constraints are required in order to generate an absolute value in a minimisation problem (Williams 1993a).

$$U_i - U_i^N \leq a_i, \forall i \quad (\text{II.10})$$

$$-U_i + U_i^N \leq a_i, \forall i \quad (\text{II.11})$$

### 4.3 Utilisation flows-based model -UFBM

The UFBM addresses the question of how to encompass sufficiently complex patient behaviour following changes in hospital supply. It uses a realistic behavioural constraint through the econometric application of the FDM, which predicts utilisation flows as a response to changes in hospital supply. The FDM-based constraint captures how the population behave in relation to their location and levels of hospital supply in their area, takes account of the process of demand for health care, and also captures the interaction between utilisation flows and supply of alternative hospitals by making use of an index for alternative supply of hospital care. The UFBM overcomes the problem of previous location studies, which have often ignored the impact of zero flows on the estimation techniques (Porell and Adams 1995).

The structure of the UFBM is designed to accommodate the characteristics of the empirical application of the FDM, which follows a TPM structure. The TPM predicts utilisation flows with two components. First, a model that predicts the probability of a population area making use of a hospital. Second, a model for predicting the level of utilisation, given that the probability of use is positive. The UFBM assumes that the probability is fixed (the impact of this assumption is discussed below) and does not change with changes in levels of supply, and uses the second part of the model to predict patients' behaviour. In addition, the choice of the better model to predict

utilisation flows does not allow for the free choice of an objective function<sup>11</sup>. This happens as the second part of the econometric model produces data in logarithms, something that has constrained the choice of objective function of the UFBM, as described just below. Similarly to the UBM, this model imposes constraints on variations in supply at the hospital level and on lower and upper limits for redistribution.

#### 4.3.1 Structure of the problem

*Objective function.* Indices in the literature aggregate one-dimensional information, such as utilisation per population area, into a composite index summarising information on predicted and normative utilisation. There is no bi-dimensional index based on population area and hospital site. Consequently, a new equity index had to be developed to sum up deviations between utilisation flows and some (more equitable) target distribution (or more precisely, differences between the natural logarithm of flows, given the behavioural constraint). This index ought to satisfy properties often cited in the literature on location of facilities and described in the literature review section (Marsh and Schilling 1994). These include: analytical tractability for problem size and computational requirements, appropriateness for interpretation, non-discrimination between the (geographic) groups being evaluated, and the principle of transfers. Following the variance of the logarithms index version presented in Marsh and Schilling (Marsh and Schilling 1994) (originally used by Theil (Theil 1967) as described below), Eq. (III.1) was created. This objective function respects the principle of transfers, with respect to utilisation flows.  $n$  and  $m$  stand for the total number of population points and for the total number of hospital points, respectively.

$$\frac{\sum_i \sum_j (\log U_{ij} - \log U_{ij}^r)^2}{n * m} \quad (\text{III.1})$$

The log nature is required as flows were generated by the TPM, for which the second part follows a log-linear structure and is used as a constraint of the MP model. The probabilities of the first part of the TPM are assumed as fixed, which allows for the natural logarithm of the flows to be divided as analysed in Eq. (III.2). The component of probabilities ( $\log \hat{p}_{ij}$ ) consists of point estimates that are fixed for the current levels of hospital supply and are not allowed to vary in the MP model. This is a restrictive assumption because when hospital capacities vary, probabilities ought to change. Nevertheless, as this is a model for redistribution, hospital capacities might vary within a limited range, which implies that changes in probabilities would be small.

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<sup>11</sup> As described below, the modelling of non-linear relations between variables with MP models is very complex, or even impossible in certain cases, therefore constraining the construction of the model.

$$\log U_{ij} = \log(\hat{p}_{ij} * U_{ij}') = \log \hat{p}_{ij} + \log U_{ij}', \forall i, j \quad (\text{III.2})$$

Variations in utilisation are compared to a distribution of utilisation formulated under an equity principle ( $\log U_{ij}^r$ ) (as seen in Eq. (III.1)). The proposed index is a measure of the dispersion of the logarithm of utilisation flows against an equity target –i.e., a measure of the variance of the logarithm. This agrees with Theil's proposition (Theil 1967)<sup>12</sup> that the variance decomposition is useful when the variable is approximately lognormally distributed. Theil has applied this in the context of study of the distribution of income. The use of the logarithm of flows as a variable in a MP model implies that it is impossible to link flows with total utilisation per population area inside the MP model. This must be taken into account in the analysis of results. As the logarithm of zero is minus infinity, areas with zero logarithmic flow took the value of one as in Eq. (III.3).

$$\log U_{ij}^r = 0 \Rightarrow U_{ij}^r = 1, \forall i, j \quad (\text{III.3})$$

In summary, the objective function minimises expression III.4: it gives the quadratic function of a linear index of the logarithm of utilisation flows. It represents the same relationship as Eq. (III.1); Eq. (III.4b) shows how the objective function is linked to the predicted values generated by the TPM.

$$\min \sum_i \sum_j \frac{b_{ij}^2}{n * m} \quad (\text{III.4})$$

$$b_{ij} = \log U_{ij} - \log U_{ij}^r = \log \left( \frac{U_{ij}}{U_{ij}^r} \right), \forall i, j \quad (\text{III.4a})$$

$$\log \left( \frac{U_{ij}}{U_{ij}^r} \right) = \log \left( \frac{\hat{p}_{ij} * U_{ij}'}{U_{ij}^r} \right) = \log \hat{p}_{ij} + \log U_{ij}' - \log U_{ij}^r, \forall i, j \quad (\text{III.4b})$$

An equity distribution target was generated in accordance with a definition of location accessibility ( $\log U_{ij}^r$ ). This target is a set of estimates of an equitable distribution of utilisation by populations. This distribution was constructed using populations going to the closest hospital by means of a MP program that minimises weighted distance between population points and hospital sites.

The use of the objective function from Eq. (III.4) requires the use of a quadratic MP model<sup>13</sup>. The quadratic function needs to be convex so that the solution of the MP program is a global (instead of local) optimum (Williams 1993a). As all the decision variables of the squared utilisation variable of expression have positive coefficients (as can be seen in the coefficients of

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<sup>12</sup> The original formula of the variance of the logarithms suggested by Theil was:  $\frac{1}{n} * \sum_i (\log U_i - \log \bar{U}^N)^2$ .

<sup>13</sup> A quadratic mathematical programming model is a model with a quadratic objective function and linear constraints.

$\log U'_{i,j}$  when taking the squares of Eq. (III.1), and given that  $\log \hat{p}_{ij}$  and  $\log U'_{ij}$  are fixed parameters), the Kuhn-Tucker conditions that guarantee global optimality are satisfied (Williams 1993b).

*Constraints.* Besides the constraint that patient behaviour must be simulated on the basis of the FDM, the UFBM uses a set of constraints that are similar to the constraints of the UBM, i.e. on the lower and upper limits of supply distribution, and on maintaining the total level of supply. The model only considers flows for which the first part of the FDM was predicted as positive. The logarithmic structure of the behavioural constraint implies that flows are linked to hospital capacity under an exponential function.

#### 4.3.2 Formulation

Decision variables:  $\log U'_{ij}$  as the logarithm of utilisation flows, given that the probability of utilisation between population  $i$  and hospital  $j$  is positive (free variable);  $D_j$  as the size of hospital  $j$  (non-negative);  $b_{ij}$  as a definitional variable for the objective function (free variable).

Parameters:  $\text{oth}\hat{e}r_{ij}$  as the parameter from the behavioural constraint that captures the impact of other factors of the FDM on hospital utilisation flows (factors unrelated to the variables of the UFBM);  $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, \hat{\alpha}_5, \hat{\alpha}_6, \hat{\alpha}_7$  as parameters estimated in the FDM, which are required to relate hospital supply and utilisation flows;  $\log \hat{p}_{ij}$  as the logarithm of the probability of a population point  $i$  making use of a hospital  $j$ ;  $\log U'_{ij}$  as the reference utilisation flow that operates as an equity target;  $\min\_D_j, \max\_D_j$  and  $D$  as defined before in the UBM;  $DumFirst_{ij}, DumSecond_{ij}$  and  $DumCentral_{ij}$  as a dummy parameter for the first hospital of use, second hospital of use and central hospital;  $DumLisboa_{ij}, DumPorto_{ij}$  and  $DumCoimbra_{ij}$  as a dummy parameter for the central hospitals of Lisboa, Porto and Coimbra.

The objective is to minimise the squared difference between predicted utilisation flows and target flows based on an equity concept, as explained above, corresponding to Eq.s (III.4) and (III.4a).

This model uses four constraints. The first constraint is on the predicted level of flows in response to changes in supply, as defined in Eq. (III.5). The formulation of this constraint has relied on the specific econometric application of the FDM developed in (Oliveira 2002). Eq. (III.5) assumes that flows depend on hospital capacity, on alternative supply, and on whether this is the closest, second closest or closest central hospital (with  $\hat{\alpha}_0 = 0.0000352$ ,  $\hat{\alpha}_1 = -0.1873067$ ,

$\hat{\alpha}_2 = 0.0000231$ ,  $\hat{\alpha}_3 = 0.0000141$ ,  $\hat{\alpha}_4 = -4.304794$ ,  $\hat{\alpha}_5 = 0.0000158$ ,  $\hat{\alpha}_6 = 0.0000255$  and  $\hat{\alpha}_7 = -0.0000136$  for the Portuguese system).

$$\begin{aligned} \log U'_{ij} = & \text{oth}\hat{r}_{ij} + \hat{\alpha}_0 * D_j + \hat{\alpha}_1 * \tilde{D}_{ij} + \hat{\alpha}_2 * DumFirst_{ij} * D_j + \\ & + \hat{\alpha}_3 * DumSecond_{ij} * D_j + \hat{\alpha}_4 * DumCentral_{ij} + \hat{\alpha}_5 * DumPorto_{ij} * D_j + \\ & \hat{\alpha}_6 * DumCoimbra_{ij} * D_j + \hat{\alpha}_7 * DumLisboa_{ij} * D_j, \forall i, j \end{aligned} \quad (III.5)$$

Second, lower and upper limits for changes in supply are imposed:

$$D_j \geq \min\_D_j, \forall j \quad (III.6)$$

$$D_j \leq \max\_D_j, \forall j \quad (III.7)$$

Third, there is a constraint on the maximum capacity in the system:

$$\sum_j D_j = D^0 \quad (III.8)$$

## 5 Results and discussion

The application to Portuguese data has been based on the following methodological choices: population numbers have been weighted by demographic need estimates generated in (Oliveira and Bevan 2003), while Euclidean distances between centroids of the *concelho* geographic units were taken as proxies for travelling costs. The UBM and the UFBM were run with a lower limit set at 80% and an upper limit set at 120% of the current capacity. Data used in this study has been based on discharges data from the diagnostic-related group (DRG) system for 1999, resident population estimates from the Portuguese National Institute of Statistics, and estimates of the FDM (Oliveira 2002). The attraction-constrained version of the gravity model was calibrated with 1999 data from the DRG database of the Portuguese system. Limited capacity was fixed at the 1999 level. The three MP models were solved with the AIMMS software package (version 3.1) (Paragon Decision Technology 2000). This section describes and compares the results of the three models, suggests further developments of the models, and describes implications of results.

### 5.1 Comparison of the three models

The results are presented in Tables 6-9 using 1999 data as reference point.

**Table 6: Utilisation impacts with a redistribution of 20% of supply both on lower and upper bound**

<i>Model</i>	<i>Total utilisation</i>	<i>Maximum utilisation rate</i>	<i>Minimum utilisation rate</i>	<i>Standard deviation utilisation rate</i>
Past data (1999)	901,229	645%	29%	0.46
DBM	901,229	100%	100%	NA
UBM	858,426	136%	37%	0.22
UFBM	675,306	327%	12%	0.48

**Table 7: Distance impacts with a redistribution of 20% of supply both on lower and upper bound**

<i>Model</i>	<i>Average distance travelled</i>	<i>Maximum average distance</i>	<i>Minimum average distance</i>	<i>Standard deviation average distance</i>
Past data (1999)	22.4	164.9	1.1	33.5
DBM	16.8	193.9	0.0	21.3
UBM	53.5	107.6	18.4	15.1
UFBM	14.6	91.94	2.2	17.0

Note: Average distances computed for all the patients from a population area

**Table 8: Number of winners and losers with a redistribution of 20% of supply both on lower and upper bound**

<i>Model</i>	<i>Number of hospital 'winners'</i>	<i>Number of hospital 'losers'</i>
DBM	42	16
UBM	29	39
UFBM	59	9

**Table 9: Comparative results from the location-allocation models**

	<i>DBM</i>	<i>UBM</i>	<i>UFBM</i>
Variation in utilisation rates	No change	+	-
Differences to needs-based distribution	+	-	+
Variations in distances travelled	+	+	+
Total distances travelled	+	-	+
Total utilisation	No change	-	-
Redistribution level	-	-	+

+ denotes positive effect; - denotes negative effect (evaluation on a comparative basis)

*DBM*: Results show an improvement in location accessibility, as measured by distance travelled (Table 7). There is no impact on total utilisation or on utilisation rates (Table 6), as the DBM assumes fixed utilisation rates per population at the small area level. Although there is a decrease in the average distance and in the standard deviation of average distances across areas, the maximum average distance from a population area is increased in the DBM (Table 7). Redistribution favours urban districts peripheral to the main urban centres. The redistribution proposed by this model is consistent with progression towards a needs-based distribution, as there is a convergence on the results for redistribution for most districts (interpreted as similar sign of variation).

*UBM*: Results show a substantial decrease in the variation of the utilisation rates (Table 6), as explicitly pursued in the objective function of this model. These gains are counterbalanced, however, by increases in distances travelled (Table 7) and by a reduction in total utilisation (Table 6). The UBM also produces substantial redistribution of hospital supply (Table 8), with a decrease in supply in central and other urban hospitals located in coastal areas and increases in hospitals in the interior and the south of the country. This is expected, as the probabilities of travelling to hospitals are based on current data and the model attempts to equalise utilisation rates by redistributing supply and utilisation so as to increase flows for hospitals with lower levels of

accessibility. The increase in distances travelled is due to the fact described above that the gravity model makes the unrealistic assumption that urban populations are prepared to travel to rural hospitals and focuses on equalisation of utilisation rates. The redistribution suggested by the UBM is incompatible with the needs-based distribution of hospital supply.

*UFBM.* The redistribution of supply by this model results in the greatest gains in geographic accessibility (Table 7). This is expected, since the equity target (included in the objective function) was based on patients having access to the closest hospital. The model proposes redistribution of supply towards peripheral hospitals in urban areas and rural hospitals in the interior and the south. In general, the proposed redistribution is consistent with progressing towards equity in terms of need of hospital care (in terms of sign of variation), except for five districts. This model reduces supply in a small number of hospitals and discriminates positively in favour of a large number of hospitals (Table 8). There is a substantial decrease in total utilisation, which might have two explanations. First, the large hospitals attract patients over large distances. When the capacity of large hospitals is decreased and that of small hospitals is increased, the reduction in flows to the central hospitals is not offset by increases in flows to smaller hospitals. This result might be partly interpreted as a reduction in supplier-induced demand. Second, the logarithmic structure of the behavioural constraint implies that decrease in the capacity of a big hospital implies a reduction in flows that is higher (in absolute value) than the increase of flows implied by an increase in capacity of a small hospital<sup>14</sup>. This is explained by the fact that the impact of hospital capacity on flows depends on the current hospital size and flows<sup>15</sup>. Moreover, redistribution of supply implies a loss in equity of utilisation per small area (as variations in utilisation rates by population area increase) (Table 6).

The UFBM is the most realistic model in that it accounts for the process of demand for hospital care and models interaction, but not so realistic in terms of reduction in total capacity. Results were found to be robust to alternative econometric formulations of the FDM; and the use of alternative equity targets –such as the use of the outputs of the DBM (instead of patients using the closest hospitals). The UFBM can be used to rank hospitals in terms of their potential for improving geographic equity of utilisation (in the case of increase in hospital capacity), by using the following sequence: a) Run the model for small levels of redistribution of supply to identify the first hospital whose capacity is to be increased; b) Put the hospital at the top in the ranking; c)

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<sup>14</sup> The district that loses most in its level of utilisation by resident populations is Lisboa.

Re-run the model with a further constraint of not changing the level of supply for that specific hospital; d) The next hospital for which an increase in supply is identified occupies the next place in the ranking; e) Repeat the sequence until all the hospitals are placed in the ranking.

Another interesting result from the UFBM is that when the upper limit for redistribution is increased (above 20%, i.e. the upper limit for changes in current capacity), the model tends to concentrate supply in a reduced number of hospitals. This result may indicate that the highest improvements in accessibility are achieved through big changes in hospital supply for such small number of hospitals. This might imply that improving equity of access and utilisation requires large changes in supply and that other policy tools might be preferred to tackle inequities in utilisation.

Comparing the results of the three models (generic results are summarised in Table 9) shows the following. First, the two models that use realistic models of patients' behaviour (UBM and UFBM) result in reductions in total utilisation. This result has not been reported in some of the previous MP models that have used behavioural information generated by gravity models (i.e. models with a structure similar to the UBM). This result shows suggests that there is a trade off between improvements in location accessibility or in equity of utilisation and total utilisation. Second, no model makes complete progress towards capitation-based allocations. Third, redistribution of supply suggested by the UFBM is intermediate in relation to the one suggested by the two other models: reduction of capacities in a small number of central hospital sites in the UFBM is combined with increases in hospitals in urban peripheral areas and in the interior and southern areas. Fourth, as expected, each model focuses on the achievement in one dimension of equity at the expense of others. The UFBM produces gains in accessibility and losses in total utilisation and in equity of utilisation; the UBM produces in equity of utilisation and losses in accessibility and in total utilisation; the DBM produces gains in locational accessibility but neglects variation in utilisation. Fifth, in comparison to past data showing wide variations in utilisation rates (Table 6): the UBM improves equity of utilisation in terms of decrease in variations in utilisation rates while the UFBM does not achieve this result (it leads to increase in variations in utilisation rates). Sixth, implementation of redistribution seems to be easiest in the UFBM as the scale of redistribution is smaller (Table 8). Finally, for Portugal, all the models point out the need to reduce hospital supply in the three districts with central provision.

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<sup>15</sup> This result can be confirmed by analysing the behaviour of the first derivative of  $U'_{ij}$  as a result of a differential variation in  $D_j$ , to be estimated by (Eq. III.5).

## **5.2 Implications for policy analysis**

The use of alternative models has illuminated the discussion of redistributive policies about hospital supply. There is neither a single objective nor optimal model to answer the redistribution problem and no single model can fully address the broad concept of equity<sup>16</sup>. Each model has both strengths and weaknesses. Results show that the pursuit of different equity objectives involves trade-offs, such as between equity of geographic access and of utilisation or between equity of geographic access and efficiency (when this is measured as total utilisation in the system). The use of more realistic assumptions about patients' behaviour makes the models increasingly complex and less transparent. The principal conclusions follow.

First, it seems important to include in analysis the assumptions about patient's behaviour. The use of 'more pragmatic' behavioural assumptions has shown that there is a trade-off between gains in geographic accessibility and gains in equity of utilisation. Losses in total utilisation are likely to be unacceptable for the policy-maker. Thus improvements in geographic equity of utilisation might be better achieved by other types of policies, such as by marginally increasing hospital supply in the system rather than by redistribution. These policies should be explored through further research.

Second, it appears that policies within the Portuguese system for improving geographic equity will result in decreases in the capacity of central urban hospitals within the system and increases in both peripheral and urban hospitals along with rural hospitals. Nevertheless, given the costs of redistribution (due to decreases in utilisation), other policies aiming to improve equity in utilisation should be explored (such as attempts to correct inequalities by marginally increasing hospital supply).

Third, even if the equity concept is specifically defined, different measurements have shown to generate different redistributive results. This shows again the need for policy-makers to clearly define the equity objectives to be pursued.

Finally, the UBFM was shown to be an alternative LAM that tackled both the problems of unrealistic assumptions about patients' behaviour with regard to hospital utilisation and of interaction between utilisation flows and supply for alternative hospitals. It has also accounted for the process of demand for hospital care. The UBFM is the best model for meeting the

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<sup>16</sup> All the models rely on a set of assumptions, some of which could be modified in future research. Discussion of this is available in (Oliveira 2003).

objectives set in this study, and it has been shown that if one aims to improve accessibility for populations located at different points, the best policy tool might be to look at alternative instruments to hospital supply. The innovative formulation of the UFBM has used an interdisciplinary approach between health economics and operational research. Health economics was used to develop realistic assumptions about patients' behaviour in the demand for hospital care. Operational research was used to provide a holistic model of the whole system, using the MP models, algorithms and software.

## **Bibliography**

- Anas, A. 1983. Discrete choice theory, information theory and the multinomial logit and gravity models. *Transportation Research -B* 17:13-23.
- Arndt. 2001. *Information Measures: Information and its Description in Science and Engineering -With 64 Figures*. Berlin: Springer-Verlag.
- Bennet, W. D. 1981. A location-allocation approach to health care facility location: A study of the undoctored population in Lansing, Michigan. *Social Science and Medicine* 15 (D):305-12.
- Cho, C.-J. 1998. An equity-efficiency trade-off model for the optimum location of medical care facilities. *Socio-Economic Planning Sciences* 32 (2):99-112.
- Church, R. L., and D. J. Eaton. 1987. Hierarchical location analysis using covering objectives. In *Spatial Analysis and Location-Allocation Models*, edited by A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company.
- Congdon, P. 2001. The development of gravity models for hospital patient flows under systems change: A Bayesian modelling approach. *Health Care Management Science* 4:289-304.
- Current, J., H. Min, and D. Schilling. 1990. Multi-objective analysis of facility location decisions. *European Journal of Operational Research* 49:295-307.
- Fang, S.-C., J. R. Kajasekera, and H. S. J. Tsao. 1997. *Entropy Optimization and Mathematical Programming* Edited by F. S. Hillier, *International Series in Operations Research and Management Science*. Massachusetts: Kluwer Academic Publishers.
- Ghosh, A., and G. Rushton. 1987. Progress in location-allocation modelling. In *Spatial Analysis and Location-Allocation Models*, edited by A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company.
- Gonzalez, M. P. L. A. 2001. Modelos multicriterio para la financiación de los Hospitales Comarcales del Servicio Andaluz de Salud. PhD Thesis, Departamento de Economía Aplicada (Matemáticas), Universidad de Málaga, Málaga.

- Goodwin, P., and G. Wright. 1998. *Decision Analysis for Management Judgment*. 2nd ed. Chichester: Wiley.
- Hallefjord, A., and K. Jornsten. 1984. Gravity models with multiple objectives -Theory and applications. *Transportation Research -B*:19-39.
- Hodgson, M. J., K.E. Rosling, and A. L. G. Storrier. 1996. Applying the flow-capturing location-allocation model to an authentic network: Edmonton, Canada. *European Journal of Operational Research* 90:427-443.
- Jornsten, K. O., and J. T. Lundgren. 1989. An entropy-based model split model. *Transportation Research -B* 23B (5):345-59.
- Love, R. F., J. G. Morris, and G. O. Wesolowsky. 1988. *Facilities Location: Models and Methods*. Edited by S. I. Gass, *Publications in Operations Research Series*. New York: North-Holland.
- Mandell, M. B. 1991. Modelling effectiveness-equity trade-offs in public service delivery systems. *Management Science* 27 (4):467-82.
- Marianov, V., and D. Serra. 2001. Hierarchical location-allocation models for congested systems. *European Journal of Operational Research* 135:195-208.
- Marsh, M. T., and D. A. Schilling. 1994. Equity measurement in facility location analysis: a review and framework. *European Journal of Operational Research* 71:1-17.
- Martin, J. J., M.P.L. Amo, R. Caballero, T. Gomez, J. Molina, and F. Ruiz. 2000. A goal programming scheme to determine the budget assignment among the hospitals of a sanitary system. In *Decision-Making: Recent Developments and Worldwide Applications*, edited by S. H. Zanakis, G. Doukakis and C. Zopounidis. Dordrecht: Kluwer Academic Publishers.
- Mayhew, L. D., R. W. Gibberd, and H. Hall. 1986. Predicting patient flows and hospital case-mix. *Environment and Planning A* 18:619-38.
- Mayhew, L. D., and G. Leonardi. 1982. Equity, efficiency, and accessibility in urban and regional health care systems. *Environment and Planning A* 14:1479-507.
- McLafferty, S. 1988. Predicting the effect of hospital closure on hospital utilization patterns. *Social Science and Medicine* 27 (3):255-62.
- Mohan, J. 1983. Location-allocation models, social science and health service planning: An example from North East England. *Social Science and Medicine* 17 (8):493-9.
- Nijkamp, P. 1978. Compromise choices in spatial interaction and regional planning models. In *Spatial Interaction Theory and Planning Models*, edited by A. Karlqvist, L. Lundqvist, F. Snickars and J. W. Weibull. Netherlands: North Holland.
- O'Kelly, M. E. 1987. Spatial-interaction-based location-allocation models. In *Spatial Analysis and Location-Allocation Models*, edited by A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company.

- Oliveira, M. 2003. Achieving Geographic Equity in the Portuguese Hospital Financing System. PhD, Operational Research Department, London School of Economics, University of London, London.
- Oliveira, M., and G. Bevan. 2003. Measuring geographic inequities in the Portuguese health care system: An estimation of hospital care needs. *Health Policy* 66 (3):277-92.
- Oliveira, M. D. 2002. A flow demand model to predict hospital utilisation. London: LSE Health and Social Care.
- AIMMS Academic (Network) - Unrestricted 3.1.468p, CONOPT Solver 2.070G (DLL). Paragon Decision Technology, Netherlands.
- Porell, F. W., and E. K. Adams. 1995. Hospital choice models: A review and assessment of their utility for policy impact analysis. *Medical Care Research Review* 52 (2):158-95.
- Rice, N., and P. Smith. 1999. *Approaches to Capitation and Risk Adjustment in Health Care: An International Survey*. Edited by Centre for Health Economics. York: Centre for Health Economics.
- Rushton, G. 1987. Selecting the objective function in location-allocation analysis. In *Spatial Analysis and Location-Allocation Models*, edited by A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company.
- Taket, A. R. 1989. Equity and access: exploring the effects of hospital location on the population served -A case study in strategic planning. *Journal of the Operational Research Society* 40 (11):1001-10.
- Theil, H. 1967. *Economics and Information Theory*. Edited by H. Theil. Volume 7 vols, *Studies in Mathematical and Managerial Economics*. Amsterdam: North Holland.
- Webber, M. 1978. Spatial interaction and the form of the city. In *Spatial Interaction Theory and Planning Models*, edited by A. Karlqvist, L. Lundqvist, F. Snickars and J. W. Weibull. Netherlands: North Holland.
- Williams, H. C. W. L., and M. L. Senior. 1978. Accessibility, spatial interaction and the spatial benefit analysis of land use -transportation plans. In *Spatial Interaction Theory and Planning Models*, edited by A. Karlqvist, L. Lundqvist, F. Snickars and J. W. Weibull. Netherlands: North Holland.
- Williams, H. P. 1993a. *Model Building in Mathematical Programming* 3rd ed. Chichester: John Wiley & Sons.
- . 1993b. *Model Solving in Mathematical Programming* Chichester: John Wiley and Sons.
- Wilson, R. M., and R. W. Gibberd. 1990. Allocating resources between health care regions: Catchment populations and allocating policy. *European Journal of Operational Research* 49:333-47.
- Wu, N. 1997. *The Maximum Entropy Method: With 53 Figures*. Edited by T. S. Huang, T. Kohonen, M. R. Schroeder and H. K. V. Lotsch, *Springer Series in Information Sciences*. Verlin: Springer.